Amirkabir Journal of Civil Engineering

Amirkabir J. Civil Eng., 53(4) (2021) 305-308
DOI:10.22060/ceej.2019.16867.6381

Using the Time-Weighted Residual Method in Forced Vibration Analysis of Timoshenko Beam under Moving Load

A. Borji, B. Movahedian Attar*, B. Boroomand

Department of Civil Engineering, Isfahan University of Technology, Iran

ABSTRACT: In this study, formulation of a recently proposed time-weighted residual method has been developed for the vibration analysis of Timoshenko beams under moving loads. Employing pre-integration relations as well as equilibrium equations is the main idea of this method. In the first step of the proposed method, the time interval is subdivided into a number of sub-intervals and then the acceleration field in each time step is defined as the combination of an unknown function and a series of exponential basis functions with constant coefficients. Finally, the solution of the problem is computed by the time-weighted residual method along with exact satisfaction of the initial and the boundary conditions at the two ends of the beam. Storing the information of solution at each time step on the exponential coefficients is the most important advantage of this method so that the solution is progressed in time without the need to discretize beams and only by using an appropriate recursive relation to update the exponential coefficients. In order to investigate the accuracy and efficiency of the proposed method, the results of solving three sample problems of constant and accelerated moving load on the beams with different boundary conditions, are compared with the results of the finite element method. This comparison illustrates the speed and accuracy of the proposed method in estimating the internal shear forces and bending moments rather than those obtained by the finite element method.

Review History:

Received: Aug. 03, 2019
Revised: Sep. 26, 2019
Accepted: Sep. 26, 2019
Available Online: Oct. 06, 2019

Keywords:

Timoshenko Beam
Moving load
Time Weighted Residual Method
Pre-Integration Relations

1. Introduction

The vibration analysis of the beams under the effect of moving loads is one of the most important study areas in the dynamic of structures. Problems such as the behavior of bridges, railways, roads, fluid transmission pipes are among these issues. Most of the researches in this field has been done based on the assumption of Euler-Bernoulli beam theory and by neglecting the effects of shear deformation and rotational inertia of the beam section. A set of analytical and semi-analytical responses for single-span and multi-span Bernoulli beams under moving load is present in [2]. The simplifying assumptions used in the estimation of analytical solutions in moving load problems have been the main reason for using and developing various numerical methods such as finite difference method, finite element method, spectral finite element method, and meshless methods.

In this paper, the idea used in [1] to solve the governing differential equation of the vibration of Timoshenko beams under moving load is developed. This method was first proposed in 2019 to solve two and three-dimensional scalar wave propagation problems and its main idea is to use pre-integration relations along with equilibrium equations. In this method, the initial conditions are fully satisfied while the equilibrium equation is satisfied through a time-weighted residual approach. The boundary conditions are also satisfied at both ends of the beam and the end of each step.

2. Methodology

The general form of the differential equation governing the vibration of the Timoshenko beam is exposed to the form of the differential equation (1) under the desired load effect.

$$\begin{cases} \kappa GA\dfrac{\partial^2 w}{\partial x^2}-\kappa GA\dfrac{\partial \varphi}{\partial x}=\rho A\dfrac{\partial^2 w}{\partial t^2}-q\left(x,t\right) \\ EI\dfrac{\partial^2 \varphi}{\partial x^2}+\kappa GA\dfrac{\partial w}{\partial x}-\kappa GA\varphi=\rho I\dfrac{\partial^2 \varphi}{\partial t^2} \end{cases} \tag{1}$$

In the above equations, w(x,t) is the transverse displacement, while φ(x,t) is the angle of rotation, q(x,t) is an arbitrary external load, Young's modulus E, shear modulus G, mass density ρ, cross-sectional area A, the moment of inertia I and the shear coefficient .

As the first step of the proposed method, the time domain is divided into some time steps. Moreover, the acceleration field in each time step is defined as the combination of an unknown function and a series of exponential basis functions with constant coefficients as follow

*Corresponding author's email: b.movahedian@cc.iut.ac.ir

$$\ddot{w}_n(x,\tau) = f_n(x) + \sum_{j=1}^{2} C_{n,j} \times \left\{ \sum_{i=1}^{M} \left(h_{j,i} \times e^{\alpha_i x + \beta_{B,i}\tau} \right) + h_{j,0} \times \tau \right\} \quad (2)$$

$$\ddot{\varphi}_n(x,\tau) = g_n(x) + \sum_{j=1}^{2} \bar{C}_{n,j} \times \left\{ \sum_{i=1}^{M} \left(l_{j,i} \times e^{\alpha_i x + \beta_{S,i}\tau} \right) + l_{j,0} \times \tau \right\} \quad (3)$$

In the next step, by satisfying the time-weighted equation of the governing equilibrium equation, the unknown function of the acceleration field is obtained. Then the constant coefficients of the acceleration field are determined by making use of the concept of source function. Finally using the pre-integration relations, a set of recurrence relations will be obtained to advance the numerical solution of velocity and displacement in time.

3. Results and Discussion

In this part, the accuracy and efficiency of TWRM solutions are investigated through a comparison between results obtained from FEM and analytical solution in [3]. The simply-supported uniform beam is traversed by q=147 N from the left to right at speed v=6 m/s. Young's modulus of the beam material is taken to be E=206GPa, shear modulus G=79GPa, and the mass density ρ=7850 kg/m^3. Assume that the length of the beam is L=2m, the width of the cross-section is b=0.1 m and the depth is h=0.025m.

The parameters used in a time-weighted residual method to solve this example include the time step, the exponential base number of source function, and the number of exponential bases of loading, are $\Delta t = 1.6\times10^{-6} s$, $M = 25$, $M' = 25$ respectively. To the velocity of the load and the length of the beam, the duration of the load is 0.33 sec.

Fig. 1 and Fig. 2 shows deflection at x=0.5L and flexural strain at x=0.25L, respectively.

According to the results shown in Fig. 1 and Fig. 2, it can be said that the accuracy of the proposed method is better than the finite element method. Increasing the accuracy of the finite element method in calculating internal forces, requires an increase in the number of elements and as a result increasing the degrees of freedom of solution. While in the proposed method without the need for the discretization of amplitude and with minimum degrees of freedom, the forceful and displaced variation responses of the beam are calculated.

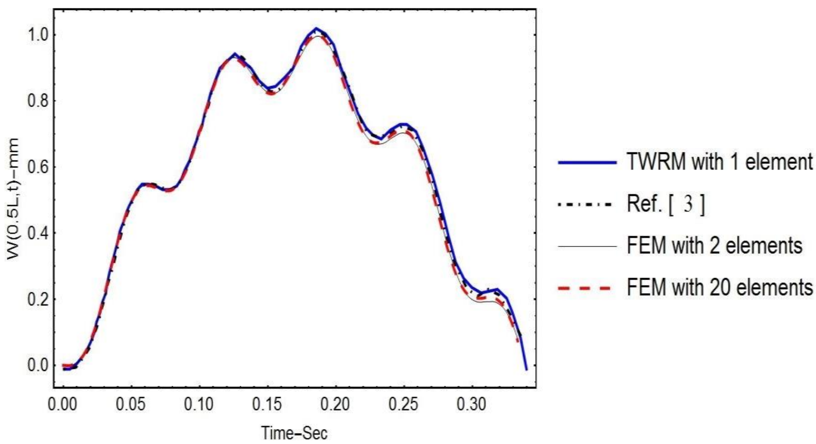

Fig. 1. Deflection at x=0.5L

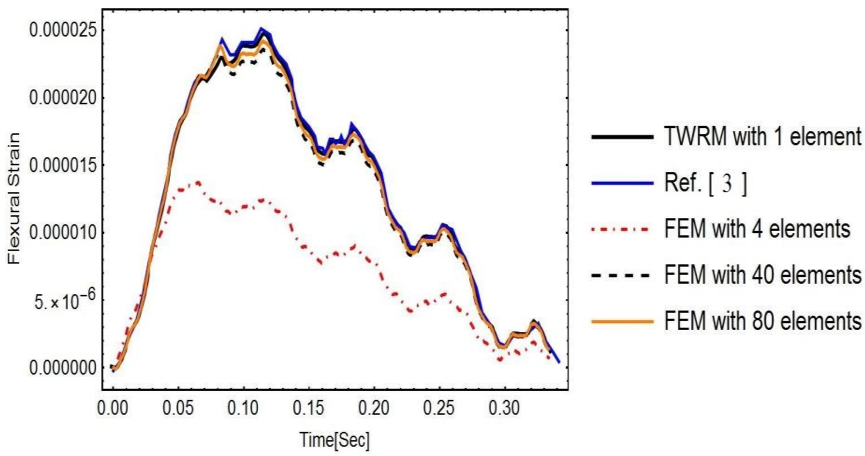

Fig. 2. Flexural Strain at x=0.25L

4. Conclusions

In this paper, the formulation of the time-weighted residual method is developed to solve the problem of the Timoshenko beam under constant and variable pace moving load. This new method makes it easy to simulate moving loads due to the independence of disassembling the beam. Therefore, the common problems of tracking the load in different member elements in existing numerical methods such as finite element have been eliminated. Another advantage of the time-weighted residual method in solving the Timoshenko beam vibration equation under moving load is the information storage of acceleration, velocity, and displacement of each time step on the exponential basis coefficients so that getting progress in solving the problem can only be possible by updating these coefficients. Moreover, the use of exponential bases with unlimited continuity order as a solution provides the accuracy of calculating the shear force and bending moment variations of the beam without increasing the degrees of freedom.

References

[1] B. Movahedian, B. Boroomand, S. Mansouri, A robust time-space formulation for large-scale scalar wave problems using exponential basis functions, International Journal for Numerical Methods in Engineering, 114(7) (2018) 719-748.

[2] J L. Frýba, Vibration of solids and structures under moving loads, Springer Science & Business Media, 2013.

[3] J.Q. Jiang, W.-Q. Chen, Y.-H. Pao, Reverberation-ray analysis of continuous Timoshenko beams subject to moving loads, Journal of Vibration and Control, 18(6) (2012) 774-784.

[4] V. Sarvestan, H.R. Mirdamadi, M. Ghayour, Vibration analysis of cracked Timoshenko beam under moving load with constant velocity and acceleration by spectral finite element method, International Journal of Mechanical Sciences, 122 (2017) 318-330.

[5] P. Lou, G.-l. Dai, Q.-y. Zeng, Dynamic analysis of a Timoshenko beam subjected to moving concentrated forces using the finite element method, Shock and Vibration, 14(6) (2007) 459-468.

HOW TO CITE THIS ARTICLE:

A. Borji, B. Movahedian Attar, B. Boroomand, Using the Time-Weighted Residual Method in Forced Vibration Analysis of Timoshenko Beam under Moving Load, Amirkabir J. Civil Eng., 53(4) (2021): 305-308.

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