



## Proposing an RC Fiber Frame Element Based on Local Stress Field Theory and Bar-Concrete Interaction

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**ABSTRACT:** This research presents an analytical model for developing a fiber frame element based on local stress field theory. The proposed formulation is developed through the Lagrangian kinematics assumption to derive the weak form of the equations in large strain conditions. In this regard, the effect of bond-slip has been considered by removing the perfect bond assumption. The governing equations for each element are developed by the directional stiffness matrix in weak form. The extracted formula is based on Timoshenko's beam theory, with axial, bending, and shear interaction effects in the domain of each element. The components of the stiffness matrix are defined through directional derivatives of the semi-linear form of the equations. Moreover, the suggested approach evolves from cubic Hermitian polynomials and the local stress field theory. The validation of the analytical method is provided by the available experimental tests. The implemented code could cover the overall behavior of reinforced concrete members, as well as, the maximum crack width, slip profile, and crack growth. The results show that such a modeling method is capable of simulating RC members.

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### 1. Introduction

Fiber-based approaches are the most commonly used due to concurrently satisfying the accuracy, reliability, computational efficiency, and robust algorithmic performance, as well as the nonlinear flexural-shear interactions consideration [1, 2]. The state-of-the-art review on the existing frame models with the inclusion of shear responses was excellently presented in [3]. Although the literature presents plenty of other interesting formulation approaches based on force-based elements [4-8] or displacement-based elements [9-11], there is still a lack of methods that consider bond-slip behaviors based on a finite strain description. Consequently, this paper is extended the displacement-based frame element of Limkatany and Spacone [12] which is different from those published previously. In other words, an exact multi-directional stiffness matrix is analytically derived based on the post-cracking bond-slip interaction between concrete and steel bars, as well as the presentation of a Timoshenko fiber frame model for large displacement analysis by using Green-Lagrange finite strain tensor. The present model is a simple applicable approach that is almost accurate and time-saving, also it holds proper convergence compared to micro modeling methods.

### 2. Methodology

The approach can be applied to finite elements consisting of fiber beam-column elements which are prepared in the MATLAB framework. The model has been constructed by using the equilibrium conditions of an infinitesimal segment

with bond interfaces at bars and constant external forces.

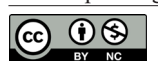
In this research, the weak formulation of updated Lagrangian (UL) kinematics is used to derive the finite element equations of a two-node Timoshenko plane beam element. The suggested approach evolves from cubic Hermitian polynomials, which has been well established by Bazoune et al. [13]. The main advantage of the developed expressions of shape functions over the classical shape functions is the shear deformation factors that can account for shear effects. Hence, the nonlinear strain vector ( $\epsilon$ -) including the axial strain ( $\epsilon_{xx}$ ) and the transverse shear strain ( $\epsilon_{yx}$ ) defined as:

$$\bar{\epsilon} = \begin{Bmatrix} \epsilon_{xx} \\ 2\epsilon_{yx} \end{Bmatrix} = \begin{Bmatrix} e - y\kappa \\ \gamma \end{Bmatrix} \quad (1)$$

$$\begin{Bmatrix} e \\ \gamma \\ \kappa \end{Bmatrix} = \begin{Bmatrix} (1 + u'_x) \cos \theta + u'_y \sin \theta - 1 \\ -(1 + u'_x) \sin \theta + u'_y \cos \theta \\ \kappa \end{Bmatrix}$$

in which, the three strain quantities ( $e$ ,  $\gamma$ ,  $\kappa$ ) characterize axial strains, shear strains, and curvatures, respectively. The bond-slip between the surrounding concrete and the  $i$ th fiber of reinforcement ( $u_{slip}$ ) and the normal strain of the concrete ( $\epsilon_{xx}^{con}$ ) and steel bar strain of  $i$ th layer ( $\epsilon_{xx}^{bars}$ ) are defined. Regarding the aforementioned aspects, the derivation of the weak form of the governing differential equations and its numerical implementation is written. To make finite element relations, a stable discretization is presented. As the local assembly proceeds, continuous displacements field discretization is applied. Thus, the element stiffness matrix

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for continuum finite elements from the governing differential equation can be derived. Considering an incremental formulation of equilibrium, the tangent stiffness matrix is obtained through the first variation of the internal force vector in each degree of freedom direction which named multi-directional stiffness matrix as:

$$\begin{aligned}
 & \{\widetilde{u}_N\}_i^T \int_L \underbrace{\lambda_N \left( \frac{\kappa}{B_w \{\widetilde{u}_w\}} \right) B_N dx}_{k_{u_N}^{u_N}} \{\Delta \widetilde{u}_N\}_i \\
 & \{\widetilde{u}_N\}_i^T \int_L \underbrace{-\lambda_N \frac{\kappa(1 + B_N \{\widetilde{u}_N\})}{\{B_w \widetilde{u}_w\}^2} B_w dx}_{k_{u_N}^{u_w}} \{\Delta \widetilde{u}_w\}_i \\
 & \{\widetilde{u}_N\}_i^T \int_L \underbrace{\begin{bmatrix} \lambda_N^s & 0 \\ 0 & \lambda_N^s \end{bmatrix} B_{slip} dx}_{k_{u_N}^{u_{slip}}} \{\Delta \widetilde{u}_{slip}\}_i \\
 & \{\widetilde{u}_w\}_i^T \int_L \underbrace{-\lambda_w \left( \frac{\kappa}{B_w \{\widetilde{u}_w\}} \right) B_N dx}_{k_{u_w}^{u_N}} \{\Delta \widetilde{u}_N\}_i \\
 & \{\widetilde{u}_w\}_i^T \int_L \underbrace{\lambda_w \frac{\kappa(1 + B_N \{\widetilde{u}_N\})}{\{B_w \widetilde{u}_w\}^2} B_w dx}_{k_{u_w}^{u_w}} \{\Delta \widetilde{u}_w\}_i \\
 & \{\widetilde{u}_w\}_i^T \int_L \underbrace{-\begin{bmatrix} \lambda_w^s & 0 \\ 0 & \lambda_w^s \end{bmatrix} B_{slip} dx}_{k_{u_w}^{u_{slip}}} \{\Delta \widetilde{u}_{slip}\}_i \\
 & \{\widetilde{u}_{slip}\}_i^T \int_L \underbrace{B_{slip}^T \lambda_{slip}^\sigma \left( \frac{\kappa}{B_w \{\widetilde{u}_w\}} \right) B_N dx}_{k_{u_{slip}}^{u_N}} \{\Delta \widetilde{u}_N\}_i \\
 & \{\widetilde{u}_{slip}\}_i^T \int_L \underbrace{-B_{slip}^T \lambda_{slip}^\sigma \left( \frac{\kappa(1 + B_N \{\widetilde{u}_N\})}{\{B_w \widetilde{u}_w\}^2} \right) B_w dx}_{k_{u_{slip}}^{u_w}} \{\Delta \widetilde{u}_w\}_i \\
 & \{\widetilde{u}_{slip}\}_i^T \left( \int_L B_{slip}^T \lambda_{slip}^\sigma B_{slip} dx + \int_L N_{slip}^T \lambda_{slip}^\tau N_{slip} dx \right) \{\Delta \widetilde{u}_{slip}\}_i \\
 & \underbrace{\hspace{10em}}_{k_{u_{slip}}^{u_{slip}}}
 \end{aligned} \tag{2}$$

### 3. Constitutive models

Under the uniaxial tension-compression or biaxial compressive stress state of the material, nonlinear material characteristics are considered (see [14], [15], and [16]). In the RC structures, the constitutive relation in the uncracked state is restricted to linear elasticity. The constitutive shear model is adapted according to Li [17] which was developed for modeling the nonlinear behavior of concrete elements. In this paper, the local stress field theory, presented by Soltani and Maekawa [18], is considered.

### 4. Numerical Results and Solution

Some numerical examples are used to verify the accuracy and show the efficiency of the proposed material nonlinear frame element as well as the solution marching schemes. The iterative-incremental method (Arc-Length method) with a variable stiffness scheme was applied to analyze structures. Afterward, several numerical investigations were performed with the proposed model to study the effects of nonlinear shear deformations and flexural responses, simultaneously. The results of the nonlinear computer analyses are compared

with the observed data and analytical results. Some examples are considered as performed tests by Gilbert and Nejadi [19], Sasani, Werner, and Kazemi [20], and Pham, Tan, and Yu [21].

### 5. Conclusions

The main characteristics of the method are substantially the flexibility formulation and the constitutive relationship characterized by a fixed smeared crack model. The proposed model was calibrated and validated through a comparison with experimental results and various numerical analyses were performed to study the influence of nonlinear flexural-shear interaction. Thereafter, this method could yield accurate and convergent results in agreement with the problems.

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