

# Reducing computational efforts in linear and nonlinear analysis of peridynamic models under impact loads

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## ABSTRACT

Peridynamic theory with a new formulation in the equations of motion, replaces the partial differential equations with integral equations. Due to this capability, it is possible to model crack initiation in any direction without the need to consider crack-growth criteria. One of the main problems in peridynamic theory is its high computational efforts due to its dynamic nature. If the critical time step of the numerical integration is greater than the loading time step, it will increase the cost of calculations. In this paper, using wavelet transform, peridynamic problems under irregular and random impact loads are analyzed. The aim of this study is to increase the computational speed for these problems. The method presented in this paper is investigated on two material models, namely Prototype brittle material and micro-plastic material. In this regard, structures with linear and nonlinear behavior have been analyzed considering the effects of discontinuities (such as cracks) and without considering the effects of discontinuities. The selected structures include two beams. Each beam is subjected to two types of irregular impact loading. The beams are analyzed once with the main impact (wave) function and once with the approximate impact (waves) functions obtained using wavelet transform. Based on the results of linear and nonlinear analyses of this study, it can be judged that the presented method reduces the computational cost by 87% in perdynamic models with linear behavior. It also bring a 94% reduction in computational costs in predynamic models with nonlinear behavior.

## KEYWORDS

Peridynamic, Wavelet transform, Cracking, Crack line, Computational efforts

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## 1. Introduction

Many problems in the mechanics of solids involve discontinuities in the geometry of the body or in the displacement field. Cracking and predicting crack growth is one of the discontinuities that challenge the solution of solids mechanics problems. Issues related to cracking and crack growth are of great importance because the presence of cracks in an object creates special conditions at the tip of the crack that cause infinite stresses. This was first investigated and proven by Griffith[1]. This led to the introduction of the theory of linear elastic fracture mechanics. In this theory, it is necessary to create a prefabricated primary crack in the body. Also in this theory, crack onset and crack growth are examined separately and separate criteria should be used to determine the direction of crack. Therefore, this method has many complications in the study of cracks. To overcome these problems, a method called peridynamic has been proposed. In 2000, Silling [2] proposed a method that could accurately analyze cracked structures. The basis of the proposed method is based on displacement and integral equations. Therefore, this method is still stable despite the discontinuities that occur in the analysis environment. The method proposed in 2005 was further developed by Silling and Askari [3]. In the proposed method, the forces between the particles are at a certain distance from each other. This distance is called the band. In this method, the onset of cracking and its growth and failure in different places is predicted with very good accuracy.

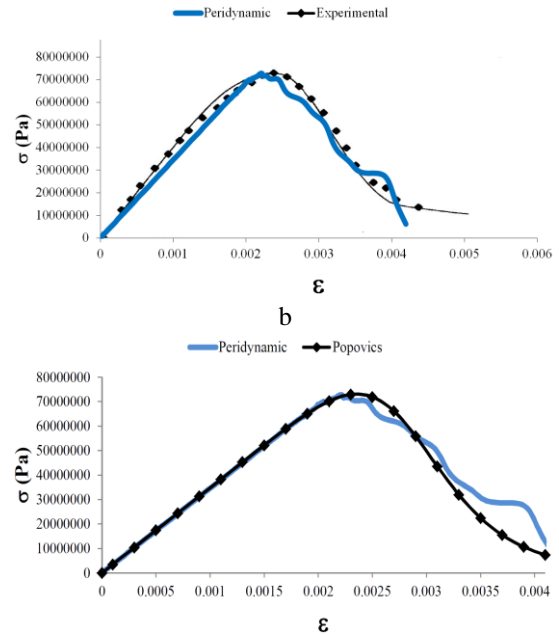
Despite all the attractive features of perdynamic models, they are often computationally more expensive than classical finite element analysis methods. In this paper, for the first time, wavelet transform is used to reduce calculations in perdynamic problems under discrete loads.

## 2. Verification

In this paper, the compressive strength test of a  $15 \times 15 \times 15 \text{ cm}^3$  cubic sample of concrete is used. The concrete strength is 73 MPa. The modulus of elasticity and the Poisson ratio are 36 GPa and 0.3, respectively [4, 5]. The results of perdynamic analysis have been validated with experimental results in reference [4]. The results are also compared with the Popovics model [6].

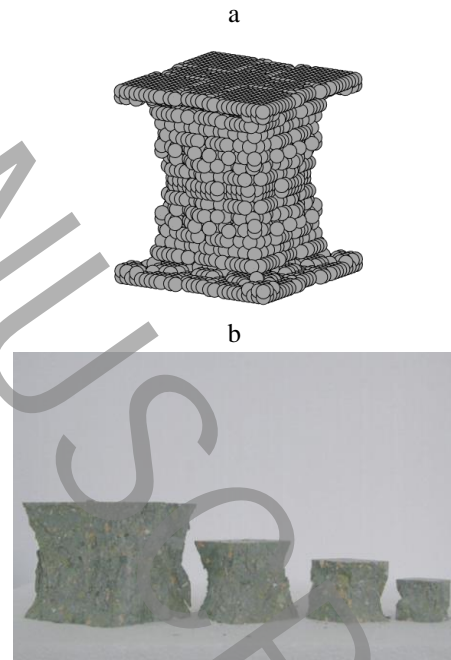
Based on Figure 1, it can be seen that the perdynamic method has a good performance for simulating concrete specimens under pressure.

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**Figure 1 Validity of a compression test. a) Validation with reference experimental results [4]. b) Validation with Popovics model**

Also in this paper, the shape of the broken specimen obtained from perdynamic theory is compared with the shape of the type of broken cubic specimens. In this regard, in Figure 2, the broken cubic samples in the laboratory [7] are compared with the cubic sample obtained from the theory of perdynamic.

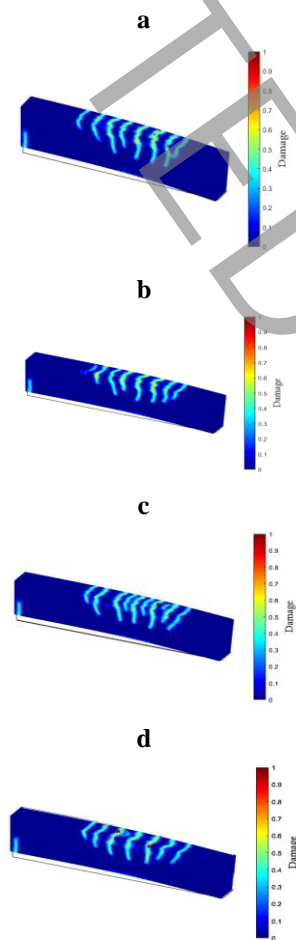


**Figure 2 Comparison of the shape of broken specimens in the laboratory with a specimen obtained from perdynamic theory. a) Obtained sample from perdynamic b) Broken samples in the laboratory [7]**

### 3. Wavelet transform in peridynamic

In this section, the effects of reducing the number of discrete wave points on peridynamic analyses results is investigated. For this purpose, the studied structures are subjected to two impact loads.

In this part, a reinforced concrete cantilever beam with longitudinal rebars with a length of 1 meter is selected. Figure 3 shows the contour related to the crack line and the amount of particle damage in the peridynamic method. According to Figure 3, it can be seen that using approximate waves does not cause much error in the crack line. This means that if the wave after four steps of approximation is used instead of the main wave, the crack line and particle damage are very similar to the case when the main wave is used.



**Figure 3** Crack line and peridynamic particle damage. a and b) Calculations using the main wave and c and d) Calculations using the approximate wave after four steps of approximation for two impact loads

### 4. Conclusion

It should be noted that due to the generally unpredictable events that occur during the nonlinearity of the structure, different approximate waves with different steps of approximation show errors which are

acceptable or not. Based on the results of case studies used in this paper it can be said that in all structures, the approximate wave at the third step of approximation with a reduction of 87% of the computation time can be a reliable approximate wave.

### 5. References

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