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Application of Multiquadric Radial Basis Function method for Helmholtz equation in seismic wave analysis for reservoir of rigid dams

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ABSTRACT: The high costs of mesh generation in mesh-dependent solution, weakness in capturing singularities, the need of modeling all over the domain, the need of problem dependent fundamental solutions, etc. are some of weaknesses in the common numerical mesh-dependent methods for solving continuum mechanics boundary value problems. In this study, aiming for eliminating some of these shortcomings, one of the well-known Radial Basis Functions (RBF) methods, Multiquadric (MQ), is developed for dynamic analysis of 2D reservoirs of rigid dams in frequency-domain. To this end, the Helmholtz equation and the governing complex boundary conditions are reproduced using MQ function in the frequency domain. The results show that with the use of real and complex forms of the MQ function, the computational time will be respectively optimized for frequencies smaller and larger than the natural frequency of the reservoir. Also, to determine the most important factors affecting both the accuracy and convergence of MQ method, first the inefficiency of some of the previously introduced methods is proved, and then a new high-speed algorithm is presented. It is shown that the optimal shape parameter for MQ method can be formulated in terms of the frequencies of seismic records. This advantage simplifies the application of MQ method in this particular problem and reduces the computational time, considerably. The high accuracy of the present method is shown in two different examples, where the effects of sediment absorption may either be considered or not. The high accuracy compared to the exact solutions achieved in this paper is due to a continuous estimation function defined all over the domain and also due to the simple algorithm used for finding the optimal shape parameter.

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Shape parameter
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1. INTRODUCTION

In order to avoid meshing and its difficulties and costs, new methods are being introduced and developed for dynamic analysis of concrete dams. In this regard, the present study develops the MQ-RBF method for the dam seismic analysis. MQ-RBF as a meshless method is more convenient and accurate than other RBF methods for solving partial differential equations (PDEs) [1, 2]. Also, this method is more efficient than the mesh-based Finite Element and Boundary Element Methods [3]. Both the accuracy and the speed of convergence of MQ-RBF depend strongly on its shape parameter. So far, researchers have been working on many methods for determining the optimal shape parameter but a comprehensive method has not been developed yet [4-6]. In this study, the commonly previous methods have been investigated for determining the optimal shape parameter and an appropriate algorithm has been presented for analyzing the reservoir of rigid dams for incoming seismic waves. The efficiency and accuracy of the present approach compared with the exact solutions have been shown through two different examples with and without considering the effects of sediment absorption.

2. METHODOLOGY

The governing PDE for distribution of frequency-domain seismic waves in reservoir of rigid dams is the Helmholtz equation:

$$\nabla^2\varphi + K^2\varphi = 0 \tag{1}$$

where φ is the velocity potential function and K is the wave number that is defined as the ratio of the excitation frequency to the velocity of sound waves in the water ($K = \omega / C$). Also, boundary conditions of the reservoir domain are defined for the reservoir, bed, dam body and water free surface, respectively, as follow:

$$\frac{\partial\varphi}{\partial n} = -\frac{i\omega}{C}\varphi \tag{2}$$

$$\frac{\partial\varphi}{\partial n} = -\frac{i\omega}{\beta C}\varphi \tag{3}$$

$$\frac{\partial\varphi}{\partial n} = \frac{\hat{a}_{ns}}{i\omega} \tag{4}$$

$$\frac{\partial\varphi}{\partial n} = -\frac{\omega^2}{g}\varphi \tag{5}$$

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where n is the normal direction to the boundary, β indicates the acoustic impedance ratio of the foundation to the reservoir, $\hat{a}_{ns}$ implies the normal component of boundary acceleration, g is the gravitational acceleration, and $i=\sqrt{-1}$.

MQ approximates the solution of 2D Helmholtz equation with the following estimation function:

$$f(x,y)=\sum_{j=1}^{N}\lambda_j\sqrt{(x-x_j)^2+(y-y_j)^2+c^2} \tag{6}$$

in which, (x_j, y_j) are the coordinates of the computational nodes, λ_j are unknown coefficients which will be obtained using N points in the computational domain and c is the shape parameter. A new high-speed algorithm is proposed in this study to select the optimal value of c [4]. Furthermore, regarding the imaginary part in the boundary conditions, the approximation function is extended from equation (6) to the following complex form

$$f(x,y)=\sum_{j=1}^{N}\lambda_{Rj}\sqrt{(x-x_j)^2+(y-y_j)^2+c_R^2}+ i\sum_{j=1}^{N}\lambda_{Ij}\sqrt{(x-x_j)^2+(y-y_j)^2+c_I^2} \tag{7}$$

In this study, it has been shown that the complex form of MQ function (Equation 7) is more accurate than equation (6) for frequencies that are more than natural frequency of the reservoir.

3. RESULTS AND DISCUSSION

In order to evaluate the proposed approach, the hydrodynamic pressure distribution has been calculated in the reservoirs of two rigid gravity dams (Figures 1-3). The analytical solutions for both examples exist. In the second example, the wave absorption effect of sediments is considered in the bottom of the reservoir, while it is not considered in the first example.

The results show that the optimal shape parameter can be formulated in terms of the frequencies of seismic records (Figure 4). This advantage simplifies considerably the application of MQ method in this particular problem and reduces the computational time.

4. CONCLUSIONS

MQ-RBF benefits from the ability for creating a continuous solution function all over the domain,

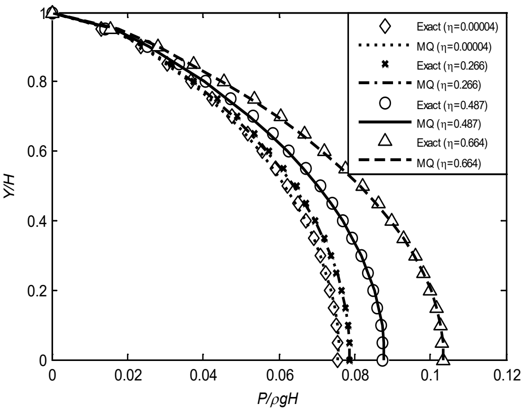

Fig. 1. Distribution of hydrodynamic pressure on dam in example 1.

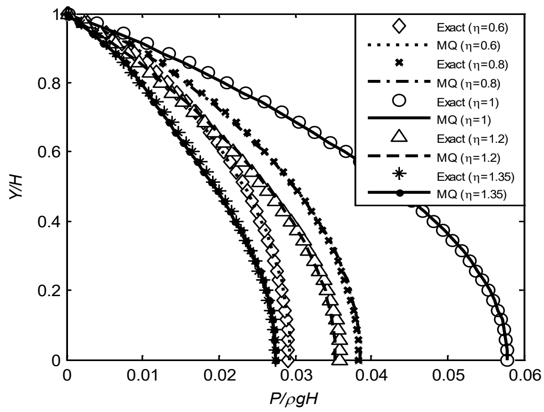

Fig. 3. Distribution of hydrodynamic pressure on dam in example 2.

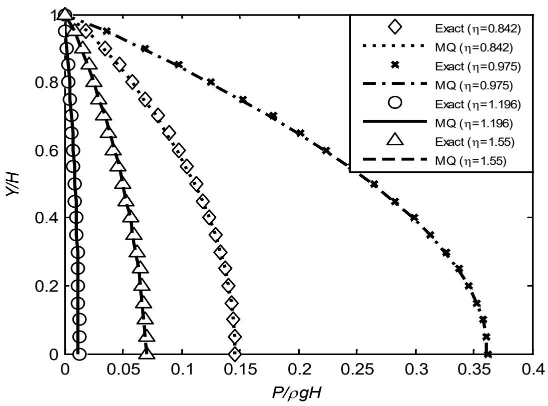

Fig. 2. Distribution of hydrodynamic pressure on dam in example 1.

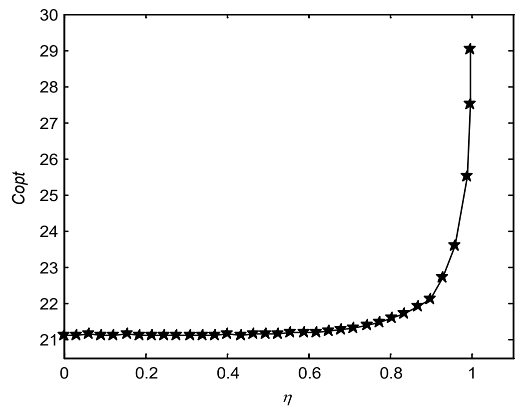

Fig. 4. Variations of optimal shape parameter in terms of frequency ratios in example 2.

independent computational points, high capability for simulating irregular and complex geometries, using domain decomposition technique for simply simulating dam-reservoir-foundation interaction problems, using the strong form of governing equations, easy generalization for 3D problems, easy to use for solving complex problems, etc. In this study, the PDEs and their complex boundary conditions governing the hydrodynamic pressure distribution in the reservoir of rigid dam have been produced for the first time, where an MQ function in the frequency domain has been used. It has been shown that the original and complex forms of this solution function are optimal in terms of accuracy and computational cost for MQ solution which depends strongly on the optimal value of the shape parameter, ten previously introduced methods have been examined and it has been found that they are not applicable for the problem considered in this paper. Subsequently, a new high-speed algorithm has been proposed for the MQ method for seismic analysis of dam reservoirs. Two different examples were solved for validation and the results show the capability and accuracy of the proposed approach compared with the exact solutions.

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