Amirkabir Journal of Civil Engineering

Amirkabir J. Civil Eng., 52(9) (2020) 563-566
DOI: 10.22060/ceej.2019.16126.6138

Study of Growth of Cohesive Crack in two Phase Environments with Extended Finite Element Method

A. Shoostari* , S. Baygi

Ferdowsi University of Mashhad, Department of Civil Engineering, Iran

ABSTRACT: Initiation and progression of cracks in a saturated porous media is an important topic which has attracted considerable attention from researchers in the recent years. Extended finite element method (EFEM) is a contemporary technique removing the necessity of consecutive meshing of the problem in the analysis process. In the EFEM by enriching the elements whose discontinuity there exists, there is no need for re-meshing at each step of the analysis. .In this paper, EFEM is used to evaluate progression of cohesive crack in a two phase saturated porous media. To analyze the saturated porous media, at the first, the equations of mass conservation, momentum conservation, and energy conservation are established to consider simultaneous effects of displacement, pressure, and temperature on the crack progression. The cohesive model is used to simulate crack progression. Heavy-side functions are used to enrich finite elements and the resulting system of equations are solved by Newton Raphson method. Finally, the numerical model were analyzed by other researchers is considered to evaluate the derived relationships. Numerical result show that maximum variation by other researchers is 5%.

Review History:

Received: 2019-04-15
Revised: 2019-06-05
Accepted: 2019-06-07
Available Online: 2019-07-04

Keywords:

Extended finite element method
meshing
saturated porous media
cohesive crack
growth of crack

1. INTRODUCTION

The extended finite element was used for the first time by Belytschko and Black in 1999 [1]. They utilized crack-tip displacement functions to enrich crack tip elements. Later, Moes et al. developed this approach and named it "extended finite element method" (XFEM) [2]. The XFEM was used to analyze two-phase problem for the first time by Belytschko and Chisa [3]. In 2007, Tethore et al, modeled cracked saturated porous medium for the first time [4]. Schrefler also proposed a new model which takes into account effect of different parameters such as displacement, temperature, capillary pressure and porous media pressure for a two-phase environment, simultaneously [5]. Vaziri proposed a finite element formulation which take into account the effects of heat and fluid flow in a multi-phase environment [6]. In the following of the mentioned studies, Pandey et al. investigated the effects of fluid injection on the crack in a non-isothermal porous medium [7].

In this study, the development of cohesive cracks in saturated two-phase environments has been investigated using the XFEM. Accordingly, first the relations of momentum, mass and energy conservations in saturated porous media are established, and then the relationships for the cohesive cracks were modified and rewrote by the matrix notation using the XFEM. Finally, to study the derived relations, the numerical model that has been developed is analyzed and evaluated using MATLAB program.

*Corresponding author's email: ashoosht@um.ac.ir

2. FORMULATION

The weak form of the momentum equation can be represented as equation 1.

$$\int_\Omega \mathrm{N}_u^T (\mathrm{S}^T \sigma + \rho_t b - \rho_t \ddot{u}) d\Omega = 0 \tag{1}$$

in which, N_u^T is the shape function of the standard finite element method. Using the part-by-part integration method and the divergence theorem, the equation (1) can be rewritten in the form of equation (2).

$$\int_\Omega B^T \sigma d\Omega + \int_\Omega N_u^T \rho \ddot{u} d\Omega = \int_\Gamma N_u^T t d\Gamma + \int_\Omega N_u^T \rho b d\Omega \\ , B = SN_u \tag{2}$$

In this equation, represent the boundary of the environment. In order to consider the crack effects, the difference of variables between the two sides of the crack should be included in the relations. Using divergence theorem and the relations of cohesive cracks, the equation of momentum can be rewritten in the following form (Eq. 3):

$$
\begin{aligned}
&\int_{\Omega} (\nabla . N_u^T)\sigma \, d\Omega + \int_{\Omega} N_u^T \rho \ddot{u} d\Omega = \int_{\Gamma} N_u^T t d\Gamma \\
&+ \int_{\Omega} N_u^T \rho b d\Omega - \int_{\Gamma_d} \langle N_u^T \rangle . t d\Gamma_d + \int_{\Gamma_d} \langle N_u^T \rangle . n_{\Gamma_d} p d\Gamma_d
\end{aligned} \tag{3}
$$

Equations 4 and 5 are the mass and energy conservation relations in the cracked porous medium.

$$
\begin{aligned}
&\int_{\Omega} \nabla N_p^T k \nabla p d\Omega + \int_{\Omega} N_p^T m B \dot{u} d\Omega + \int_{\Omega} N_p^T \dot{p}[\frac{1-n}{K_s} + \frac{n}{K_f}] d\Omega \\
&- \int_{\Omega} N_p^T \beta \dot{T} d\Omega = \int_{\Gamma} N_p^T q_f d\Gamma - \int_{\Gamma_d} \langle N_p^T n_f Q_f \rangle d\Gamma_d
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
&\int_{\Omega} \nabla N_\theta^T k_{eff} \nabla T d\Omega + \int_{\Omega} N_\theta^T \{(\rho c)_{eff} \dot{T} + \rho_f c_f k \\
&(-\nabla p + \rho_f g)\} . \nabla T d\Omega = \int_{\Gamma} N_\theta^T q'' d\Gamma - \int_{\Gamma_d} \langle N_\theta^T Q'' \rangle d\Gamma_d
\end{aligned} \tag{5}
$$

Equation 6 is derived by transforming momentum relation into the matrix notation, using the XFEM.

$$
\begin{aligned}
&\int_{\Omega} B^T \sigma' d\Omega - \int_{\Omega} B^T \alpha m p d\Omega + \int_{\Omega} N_u^T \rho \ddot{u} d\Omega + F_{int} = F_{ext} \\
&F_{int} = (\int_{\Gamma_d} (N_u^{enr})^T n_{\Gamma_d} d\Gamma_d) \overline{p} - \int_{\Gamma_d} (N_u^{enr})^T . t d\Gamma_d \\
&F_{ext} = \int_{\Gamma} N_u^T t d\Gamma + \int_{\Omega} N_u^T \rho b d\Omega
\end{aligned} \tag{6}
$$

Which, N_u^{enr} is the enriched displacement shape function. The matrix forms of the mass and energy conservation relations using the XFEM are presented by Equations 7 and 8.

$$
\begin{aligned}
&\int_{\Omega} \nabla N_p^T k \nabla p d\Omega + \int_{\Omega} N_p^T m B \dot{u} d\Omega + \\
&\int_{\Omega} N_p^T \dot{p}[\frac{1-n}{K_s} + \frac{n}{K_f}] d\Omega + q_{ine} = q_{ext} \\
&q_{int} = \int_{\Gamma_d} (N_p^{enr})^T n_f Q_f n_{\Gamma_d} d\Gamma_d
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
&\int_{\Omega} \nabla N_\theta^T k_{eff} \nabla T d\Omega + \int_{\Omega} N_\theta^T \{(\rho c)_{eff} \dot{T} + \\
&\rho_f c_f k(-\nabla p + \rho_f g)\} . \nabla T d\Omega + Q_{int} = Q_{ext} \\
&Q_{int} = \int_{\Gamma} N_\theta^T q'' dT, \; Q_{ext} = \int_{\Gamma_d} (N_\theta^{enr})^T Q'' n_{\Gamma_d} d\Gamma_d \\
&q_{ext} = \int_{\Gamma} N_p^T q_f d\Gamma + \int_{\Omega} N_p^T \beta \dot{T} d\Omega
\end{aligned} \tag{8}
$$

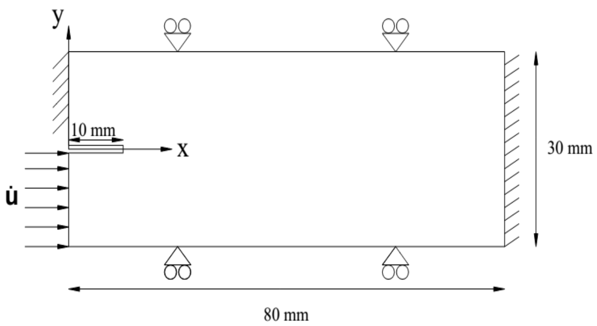

Fig. 1: geometry for model investigation.

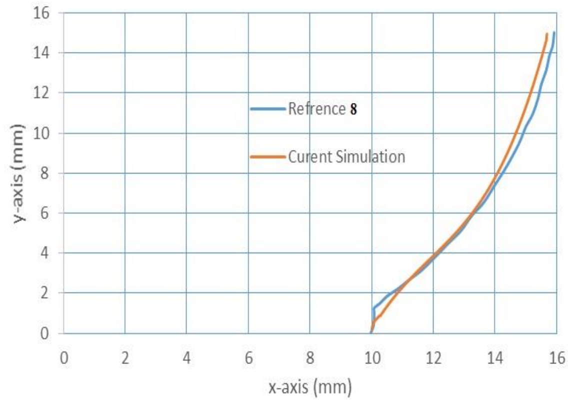

Fig. 2: Comparison between the investigation model and the reference 8.

3. NUMERICAL MODEL

In this section, for verification of the obtained relations, the numerical model suggested by Irzal is developed [8]. This model that is discussed in details in reference 8, evaluates a beam with a symmetrical discontinuity with the length of 10 mm, which is shown in Fig. 1. Fig. 2 shows the crack extension obtained using the developed MATLAB code as well as the reported results in reference number 8.

4. CONCLUSIONS

In this study, the extension of cohesive cracks in a two-phase environment was studied using the XFEM. First, in order to consider the effects of pressure, displacement and temperature, the equations of mass, momentum, and energy conservation were developed for the two-phase environment. Then, the obtained equations were discretized in the time domain. In addition, the Heavy-side function was used to enrich the cracked elements and the Newton-Raphson method was utilized to solve the relationships, simultaneously. In order to verify the obtained formulations, a numerical model that was developed by other researchers was analyzed using the suggested formulation. The obtained results showed the accuracy of the developed formulation, so that the maximum difference between the results of the model and reference 8 is 5%.

REFERENCES

[1] T. Belytschko, T.Black, Elastic crack growth in finite

element with minimum remeshing, International Journal of Numerical Methods in Engineering, 45 (1999) 601-620.

[2] N. Moes, J. Dolbow, T. Belytschko, A finite element method for crack growth without remeshing, International Jounal of Numerical Methods in Engineering, 46 (1999) 131-150.

[3] J. Chessa, T. Belytschko, An extended finite element method for two phase fluids, J Appl Mech, 70 (2003) 10-17.

[4] J. Rethore, R.d. Borst, M.A. Abellan, A two-scale approach for fluid flow in fractured porous media, Computer Methods in Applied Mechanics and Engineering, 71 (2007) 780-800.

[5] B.A. Schrefler ,X.Y. Zhan, L. Simoni, A coupled model for water flow, air flow and heat flow in deformable porous media, Int J Numer Meth Heat Fluid Flow, 5 (1995) 531-547.

[6] H. Vaziri, Theory and application of a fully coupled thermo-hydro-mechanical finite element model, Compos Struct, 61 (1996) 131-146.

[7] S.N. Pandey, A. Chaudhuri, S. Kelkar, A coupled thermo-hydro-mechanical modeling of fracture aperture alteration and reservoir deformation during heat extraction from a geothermal reservoir, Geothermics, 65 (2017) 17-31.

[8] F. Irzal, J.C. Remmers, M. Huyghe, R.Brost, A large deformation formulation for fluid flow in a progressivel fracturing porous material, Comput. Methods Appl. Mech. Engrg, 256 (2013) 29-37.

HOW TO CITE THIS ARTICLE

A. Shoostari, S. Baygi, Study of Growth of Cohesive Crack in two Phase Environments with Extended Finite Element Method, Amirkabir J. Civil Eng., 52(9) (2020) 563-566.

DOI: 10.22060/ceej.2019.16126.6138