Amirkabir Journal of Civil Engineering

Amirkabir J. Civil Eng., 52(7) (2020) 407-410
DOI: 10.22060/ceej.2019.15464.5962

# Fluid-structure interaction due to water-hammer in a pressurized pipeline considering geometrical non-linear behavior of the pipe wall

MohammadAli Mashak[1,*] , Alireza Keramat[2]

[1] MSc Student, Jundi-Shapur University of Technology, Dezful, Iran
[2] Assistant Professor, Jundi-Shapur University of Technology, Dezful, Iran

**ABSTRACT:** The research investigates a fluid filled pipeline that is connected to a tank at its upstream and to a valve in the downstream and undergoes forces of water hammer due to sudden closure of the valve. The aim is to study the possibility of instability in this pipeline when there are large lateral displacements with small strains. As conventional dynamic analysis models of beams which are based on the infinitesimal strain theory (ε=∂u⁄∂x) cannot reflect the effect of large lateral displacements, in this study axial stresses are modeled as linear stresses and strains are modeled by so called von Karman nonlinear strains. The resulting partial differential equations are solved in the time domain by the finite elements method. The linearized equation of lateral vibration is made dimensionless and then it is solved in the frequency domain so as to plot dimensionless frequencies versus the dimensionless fluid velocities which represent the stability of the pipeline. The results provides useful diagrams to anticipate possible pipeline instability induced by fluid velocity.

**Review History:**

Received: 2019-01-07
Revised: 2018-10-04
Accepted: 2019-02-27
Available Online: 2019-03-18

**Keywords:**

Fluid-structure interaction
Non-linear finite element
Structural stability
Frequency domain solution
Water hammer

## 1. INTRODUCTION

When a fluid flows into a closed conduit, an amount of pressure much larger than the atmosphere pressure is established. A pressurized pipe just like a loaded column is vulnerable to the buckling [1]. Likewise, the water hammer causes a huge pressure that can potentially destabilize the pipe structure.

The simplest pipe configuration to study this phenomenon is a reservoir-pipe-valve system. The objective is to study the structural instability due to fluid-structure interaction (FSI) caused by a sudden closure of the valve considering large transvers deflections and yet small strains.

The mathematical model includes hydraulic and structural equations. The classical water hammer theory which leads to Joukowski formula is a primary value to quantify the hydraulic loads [2]. The structural equations are the axial and lateral vibration equations. The so called Euler's buckling load is a result of the structural equations.

In 1744 Euler obtained a formula which predicts the minimum load under which a sufficiently slender perfect elastic column would buckle prior to a material failure [1]. Bazant [1] proved that a pipe under enough hydrostatic pressure does buckle. The corresponding buckling pressure is calculated using the Euler load which is imposed due to fluid pressure.

Paidoussis authored a comprehensive book about the stability of pressurized vessels. He provided a dimensionless form of vibration equation to draw frequency domain diagrams that show different regions of instability as the fluid velocity increases [3]. Later on, Lee and Chung developed Paigdoussis equations by making use of the general Lagrangian strains widely known as von Karman strains to consider the effect of geometrical non-linearity of the pipe. In this approach the destabilizing agent is fluid velocity, nevertheless, the issue of water hammer is not addressed [4].

To consider the effect of geometrical non linearity in a case of waterhamer, new nonlinear axial and lateral vibration equations are introduced and solved by finite element method (FEM) in the time domain. The results are compared with those of the linear frequency domain solutions.

### Mathematical model

Waterhammer and pipeline-vibration equations are essential in modelling FSI. Their frictionless form reads [5]:

$$\frac{\partial U}{\partial x}+\frac{g}{c_f^2}\frac{\partial H}{\partial t}-2\upsilon\frac{\partial \dot{\xi}}{\partial x}=0, \tag{1}$$

$$\frac{\partial U}{\partial t}+g\frac{\partial H}{\partial x}=0 \tag{2}$$

in which $x$ is pipe axis direction, $t$ is time, $g$ is the gravitational acceleration, $\dot{\xi}$ is the axial pipe velocity, $U$ is the flow velocity, $H$ is the pressure head, $c_f$ is pressure wave

*Corresponding author's email: m.mashak@jsu.ac.ir

speed, and $\upsilon$ is Poisson's ratio. The term $2\upsilon\,\partial\dot{\xi}/\partial x$ couples the flow hydraulics with the dynamic behavior of the pipe wall.

Taking a portion of the pipeline with length $dx$ subject to internal axial force ($N$), shear force ($V_s$) and moment $M$, leads the equilibrium equations to

$$\rho_p A_p \frac{\partial^2 \xi}{\partial t^2} - EA_p \frac{\partial}{\partial x}\left(\frac{\partial \xi}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) - \frac{A_p \upsilon D}{2e}\rho g \frac{\partial H}{\partial x} = 0 \tag{3}$$

$$EI_p \frac{\partial^4 w}{\partial x^4} + \left(\rho_p A_p + \rho_f A_f\right)\frac{\partial^2 w}{\partial t^2} - EA_p\left(\frac{\partial^2 \xi}{\partial x^2}\frac{\partial w}{\partial x} + \frac{\partial \xi}{\partial x}\frac{\partial^2 w}{\partial x^2} + \frac{3}{2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2\right) = 0. \tag{4}$$

in which $A_p, \rho_p, e, D, w$ are cross sectional area, density, wall thickness, inner pipe diameter and lateral displacements of the pipeline, respectively. The axial force ($N$) and bending moment ($M$) are

$$N = EA_p\left(\frac{\partial \xi}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) \tag{5}$$

$$M = -EI\frac{d^2 w}{dx^2} \tag{6}$$

where $I$ is second moment of inertia of the pipe cross section. Note that the term $(\partial w/\partial x)^2$ in Eq. (5) is the primary cause for the nonlinear terms in Eqs. (3), (4). Equations (3) and (4) govern the pipe vibrations taking into account geometrical nonlinearities stemming from moderate displacements and small deformations.

## Finite Element Method (FEM)

The FEM is exploited to solve the structural Eqs. (3), (4) via the following matrix representation [6, 7].

$$\begin{aligned} &M_{ij}^{11}\ddot{\xi}_j + K_{ij}^{11}\xi_j + K_{iJ}^{12}\Delta_J - F_i^1 = 0 \\ &M_{IJ}^{22}\ddot{\Delta}_J + K_{Ij}^{21}\xi_j + K_{IJ}^{22}\Delta_J - F_I^2 = 0 \end{aligned} \tag{7}$$

in which the matrix indices are: $i, j = 1,2;\; I, J = 1,2,3,4$ and $\Delta_J = \{w_1 \quad \theta_1 \quad w_2 \quad \theta_2\}^{\mathrm{T}}$ represents the transverse and slope at either sides of the element. The superscripts indicate sub-matrices and sub-vectors. The mass sub-matrices

$$\begin{aligned} &M_{ij}^{11} = \rho_p A_p \int_{x_1}^{x_2} \psi_i^a \psi_j^a \, dx \\ &M_{IJ}^{22} = \left(\rho_f A_f + \rho_p A_p\right)\int_{x_1}^{x_2} \psi_I^b \psi_J^b dx \end{aligned} \tag{8}$$

and stiffness sub-matrices [7]

$$\begin{aligned} &K_{ij}^{11} = EA_p \int_{x_1}^{x_2} \frac{d\psi_i^a}{dx}\frac{d\psi_j^a}{dx}dx \\ &K_{iJ}^{12} = \frac{EA_p}{2}\int_{x_1}^{x_2} \frac{\partial w}{\partial x}\frac{d\psi_i^a}{dx}\frac{d\psi_J^b}{dx}dx \\ &K_{Ij}^{21} = EA_p \int_{x_1}^{x_2} \frac{\partial w}{\partial x}\frac{d\psi_I^b}{dx}\frac{d\psi_j^a}{dx}dx \\ &K_{IJ}^{22} = EI_p \int_{x_1}^{x_2} \frac{\partial w}{\partial x}\frac{d^2\psi_I^b}{dx^2}\frac{d^2\psi_J^b}{dx^2}dx + \frac{EA_p}{2}\int_{x_1}^{x_2}\left(\frac{\partial w}{\partial x}\right)^2 \frac{d\psi_I^b}{dx}\frac{d\psi_J^b}{dx}dx \end{aligned} \tag{9}$$

and force sub-vector

$$\begin{aligned} &F_i^1 = \frac{A_p \upsilon D}{2e}\rho g \int_{x_1}^{x_2}\left(\frac{\partial H}{\partial x}\right)\psi_i^a dx, \\ &F_I^2 = \int_{x_1}^{x_2} q(x)\psi_i^b dx, \end{aligned} \tag{10}$$

can be found using Galerkin's weighted-residuals approach. In these relations, $\psi^a$ and $\psi^b$ indicate linear Lagrange and Hermite cubic interpolation functions employed to approximate axial and bending displacements. The $q(x)$ is external distributed transvers load which is in this case equal to zero.

## Frequency Domain Analysis

The linear form of flow induced pipe vibration yields [3]:

$$EI\frac{\partial^4 w}{\partial x^4} + MU^2\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x\,\partial t} + (M+m)\frac{\partial^2 w}{\partial t^2} = 0 \tag{11}$$

in which $M$ is fluid mass, $m$ is pipe mass and $U$ is fluid velocity. The first and the last terms are flexural and inertial effects. The second term is associated with centrifugal forces which attributes to the fluid flow in curved portions of the pipe (considering the deformed pipe shape) and the third term is associated with Coriolis effects. The dimensionless equation is [3]:

$$\frac{\partial^4 \eta}{\partial \varsigma^4} + u^2\frac{\partial^2 \eta}{\partial \varsigma^2} + 2\beta^{\frac{1}{2}}u\frac{\partial^2 \eta}{\partial \varsigma \partial \tau} + \frac{\partial^2 \eta}{\partial \tau^2} = 0 \tag{15}$$

where $u$ is dimensionless fluid velocity,

$$u = \left(\frac{M}{EI}\right)^{1/2} LU \tag{16}$$

and $L$ is the length of pipeline. Other non-dimensional quantities are

$$\beta = \frac{M}{M+m}, \varsigma = \frac{x}{L}, \eta = \frac{w}{L}, \quad \tau = \left[\frac{EI}{M+m}\right]^2 \frac{1}{L^2} \tag{17}$$

To solve Eq. (15) in the frequency domain, use is made of Fourier Transform so as to find the corresponding ordinary differential equation whose coefficients of the characteristic equation are $\alpha_i,\, i = 1,2,3,4.$ The natural frequencies of the system corresponds to the non-trivial solution of the system of equations constructed by the boundary conditions.

For the clamped-clamped supports boundaries, and $\beta$ = 0.1 the dimensionless natural frequency diagrams for both real and imaginary values (Figs. 1, 2) are determined by satisfying the following equation

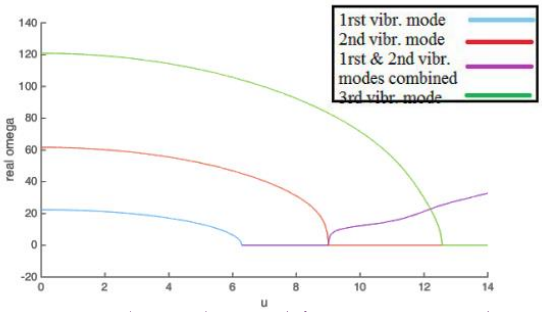


**Fig. 1. Dimensionless real natural frequency versus dimensionless fluid velocity for β =0.1 and clamped-clamped pipeline**

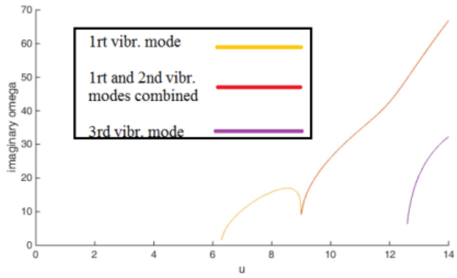


**Fig. 2. Dimensionless imaginary natural frequency versus dimensionless fluid velocity for β =0.1 and clamped-clamped pipeline**

**Table 1.The properties of the pipeline according to the case study in [8].**

| | |
|---|---|
| Length (m) | 20 |
| Diameter (mm) | 797 |
| Thickness (mm) | 8 |
| Pipe density (kg/m3) | 7900 |
| Poisson ratio | 0.3 |
| Darcy-Weisbach coefficient | 0 |
| *Reservoir head (m) | 0 |
| *Number of sections in simulation | 200 |
| *Steady state velocity (m/s) | 1 |
| *Young's modulus (Pa) | 2.1E11 |
| *Wave speed (m/s) | 1024.7 |
| *Valve closure duration (s) | 0 |

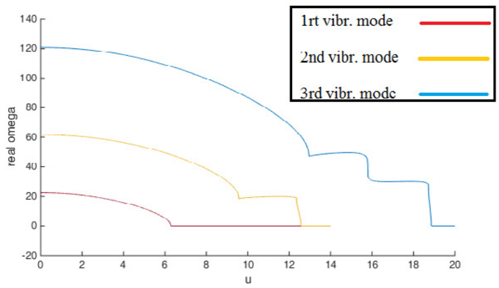


**Fig. 3. Dimensionless real natural frequency versus dimensionless fluid velocity for β =0.75 clamped-clamped pipeline**

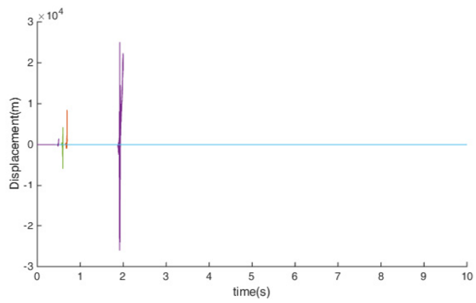


**Fig. 4. Lateral-middle point displacement subject to waterhammer.**

$$\Delta \equiv \begin{vmatrix} 1 & 1 & 1 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ e^{i\alpha_1} & e^{i\alpha_2} & e^{i\alpha_3} & e^{i\alpha_4} \\ \alpha_1 e^{i\alpha_1} & \alpha_2 e^{i\alpha_2} & \alpha_3 e^{i\alpha_3} & \alpha_4 e^{i\alpha_4} \end{vmatrix} = 0 \tag{18}$$

As seen in Fig.1 and 2, the first instability point is for $u$=2$\pi$ where the real dimensionless natural frequency of the first vibrational mode vanishes but the imaginary one arises. This instability is of buckling type. At $u$=8.99 the real part for second vibrational mode disappears. Here the so called flutter comes to effect as the first and second vibrational modes have both real and imaginary parts which are of identical pairs. Other frequency domain instability points are seen in the diagrams.

## DISCUSSION AND RESULTS

The reservoir-pipeline-valve system known as Delft Hydraulics Benchmark Problem A introduced by Tijsseling [8] is solved to investigate FSI using the prepared solver. The system specifications are given in Table 1.

The geometry information of the pipe system indicates that $\beta$ =0.75, hence the non-dimensional frequany diagarms are plotted in Figs 3, and 4.

By comparing the diagrams for the two quantities of

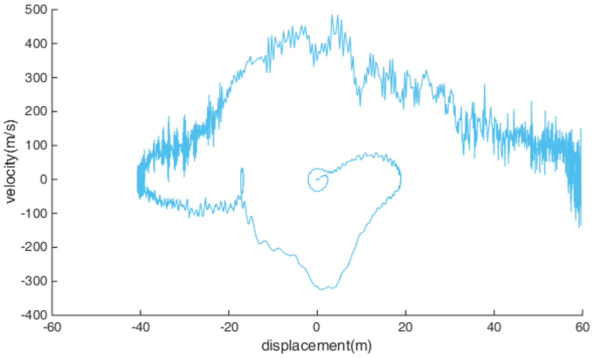


**Fig 5. Lateral-middle point displacement versus velocity subject to waterhammer for 200 m pipeline length**

$$\left(V_0 = 8\text{m/s}; E = 2.1\times10^{11}\text{ Pa};\right)$$

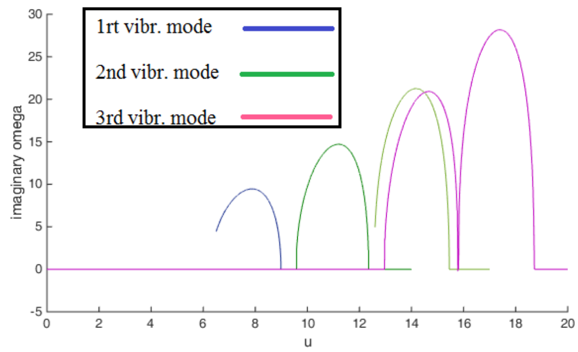


**Fig. 6. Dimensionless imaginary natural frequency versus dimensionless fluid velocity for β =0.75 clamped-clamped pipeline**

$\beta$ (0.1; 0.75), two conclusions can be drawn. Firstly, the instability points and consequently the critical fluid velocity is independent of $\beta$. Secondly, there is a restablization zone roughly in the range 9<$u$<10 where there is no imaginary natural frequency (it did not exist in the frequency diagrams for $\beta = 0.1$).

The same investigation for a pinned-pinned supports boundaries reveals that the first instability point is equal to $u=\pi$. This finding supports the results of the time-domain solution with the details depicted in Fig. 5 which is for L=20m. For u< $\pi$, the time domain results does not rise up and so the system vibrates in a stable manner (blue curve). But for higher values of velocity divergent motions emerge (green, purple and orange curves). A typical instability for a pipe of length $L$ =200 m can also be found from the time domain solutions when displacement versus velocity is shown (Fig. 6).

## CONCLUSIONS

The stability of a pipeline subject to the waterhammer load can be conducted when a geometrical nonlinear approach for the pipe vibration is adopted. The nonlinear structural analysis revealed that a pinned-pinned beam shows instabilities of buckling or flutter type provided that the dimensionless velocity exceeds a specific quantity. This quantity can be found via a frequency domain investigation when the nonlinear terms of governing equations are omitted. Then they are transformed in the frequency domain and solved so as to find the corresponding instability points.

## REFERENCES

[1] Bazant, Z.P., Cedolin, L., 1991, "Stability of Structures, Elastic, Inelastic, Fracture, and Damage Theories", Oxford University Press.
[2] Joukowski, N.E., Mem. Imperial Academy Soc. Of St. Petersburg, Vol. 9, no.5, 1898, 1900 (in Russian, translated by O. simin, proc. Amer. Water works Assoc., Vol. 24, 1904, pp. 341-424).
[3] Paigdoussis M. P. 1998 Fluid-Structure Interactions, Vol. 1: Slender Structures and Axial Flow. San Diego, CA: Academic Press Inc.
[4] Lee, Soo Il, and J. Chung. "New non-linear modelling for vibration analysis of a straight pipe conveying fluid." Journal of sound and vibration 254.2 (2002): 313-325.
[5] Wylie, E.B. and Streeter, V. L., 1993, Fluid Transients in Systems, Englewood Cliffs, New Jersey, USA: Prentice Hall.
[6] Reddy, J.N., 2004, "An Introduction to Nonlinear Finite Elements Analysis", Oxford University Press, UK, ISBN 0-19-852529-X, 978-0-19-852529-5.
[7] Keramat, A., Ahmadi, A., 2012, "Axial vibration of viscoelastic bars using the finite-element method", Journal of Engineering Mathematics, 77, 105-117.
[8] Tijsseling, A.S., 2003, "Exact solution of linear hyperbolic four-equation system in axial liquid-pipe vibration". Journal of Fluids and Structures 18, 179–196.

**HOW TO CITE THIS ARTICLE**

*M.A. Mashak, A.R. Keramat, Fluid-structure interaction due to water-hammer in a pressurized pipeline considering geometrical non-linear behavior of the pipe wall, Amirkabir J. Civil Eng., 52(7) (2020) 407-410.*

**DOI:** 10.22060/ceej.2019.15464.5962