Operation of the non-linear Muskingum model in the prediction of the pollution breakthrough curves through the river reaches

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ABSTRACT

The Muskingum model in both types of the linear and non-linear is one the most common models in the flood routing through the river reaches. The simplicity and being stepwise in the calculation of the exit flood hydrographs are the advantages of this model. Because of the similarity between the shape of the flood hydrograph and pollution breakthrough curves, it is tried to examine the applicability of the non-linear Muskingum model in the prediction of the contaminant concentration in downstream of the river reaches. The field data series of the MONOCACY and ANTIETAM Creek Rivers which were gathered by USGS have been used. During the tests, Rhodamine were used and the concentration pollute graphs were acquired in the four and eight cross sections of the mentioned rivers, respectively. The triple sets of the model parameters have been extracted in each reach, then the BC curves have been simulated in each position using them. It is observed that this model can rebuild the dimensions of the exit BC curve properly but, it also has some limitations in the modeling of the convection term of the pollution using average flow velocity. For its solution, the extracted BC curves have been transported along the time axis with magnitude of $\frac{L}{u}$ in which L is reach length and u is the average flow velocity. Also, for better understanding of the effects of the model parameters in the simulated concentrations, the sensitivity analysis has been performed and it is found that the parameters of the m, k and x are the most to less effective parameters in the concentration calculation, respectively. It was also found that the power parameter of this model (m) for pollution transport fluctuates in the range (0.1-1.4) and has an average value of 0.85. The value of the weighted coefficient (x) was also obtained in the range (-1 to +1), but the frequency of values greater than zero was greater and its average value reduced to 0.91.

KEYWORDS

River, Pollution, non-linear Muskingum, Flood, breakthrough curve

1. Introduction

Flood prediction in different parts of the river reaches is very important in terms of the design of inline and lateral structures and utilizing river discharge. Having the capability of flood prediction helps to reach neighbors to have technical and economic solutions for flood events [1]. The Muskingum method is one of the most basic hydrological methods of flood prediction which is developed by McCarty [2] on the Muskingum River. One of the most important challenges of this model is to determine its parameters. It has two parameters in the linear and three parameters in the nonlinear model. Previous researchers have proposed a wide variety of methods for using and parameter estimation of linear and nonlinear mode [3- 6]. Additionally, the provision of qualitatively reliable water resources is also one of the main challenges in the operation of river flows. Many chemical parameters affect the quality of river water flow and affect their self-purification properties [7]. Due to the similarity of flood hydrographs and contaminant breakthrough curves, attempts have been made to use the Muskingum method in pollution concentrations prediction through the river reaches. In the current study, it was tried to examine the accuracy of Muskingum model in concentration prediction through the different parts of the river reaches.

2. Materials and Methods

2.1. Model Definition

Generally, the linear Muskingum model is represented by continuity and storage relationships according to Eqs. (1-2) through the natural rivers.

$$\frac{dS_t}{dt} = I_t - Q_t \tag{1}$$

$$S_t = k\left[xI_t + (1-x)Q_t\right] \tag{2}$$

In which, S_t water storage through the river channel at time t, I_t and Q_t are the inflow and outflow to the river reach, respectively. k is the storage parameter of the reach, and x is the weighted coefficient.

The linear Muskingum model is usually less accurate in predicting floods; Therefore, the nonlinear Muskingum model (Eq. 3) is used to increase the estimation accuracy.

$$S_t = k\left[xI_t + (1-x)Q_t\right]^m \tag{3}$$

Where, m is a power parameter that defines the degree of nonlinearity of the river reach.

To estimate the output flow, Eq. 3 is rewritten and presented as Eq. 4 based on the output flow.

$$Q_t = \left(\frac{1}{1-x}\right)\left(\frac{S_t}{K}\right)^{\left(\frac{1}{m}\right)} - \left(\frac{x}{1-x}\right)I_t \tag{4}$$

Also, by rewriting the continuity relationship (Eq. 1 and Eq. 4), Eq. 5 is obtained for the storage rate through the river reach.

$$\frac{\Delta S_t}{\Delta t} = -\left(\frac{1}{1-x}\right)\left(\frac{S_t}{K}\right)^{\left(\frac{1}{m}\right)} + \left(\frac{1}{1-x}\right)I_t \tag{5}$$

Where $\frac{\Delta S_t}{\Delta t}$ is equal to the rate of change of storage, ΔS_t is the flow storage change, and Δt time interval which is based on the concepts of the kinematic wave must be greater than $2kx$.

In this study, to rewrite the flow parameters based on the input and output concentration parameters, Eqs. 1-5 have been rewritten by replacing CI_t ، CQ_t ، CS_t , ΔCS_t with I_t ، Q_t ، S_t ، ΔS_t parameters, respectively and the model coefficients (x, k and m) remain unchanged.

By application of logarithm operator to Eq. 3 and rewriting its parameters, Eq. 6 is obtained.

$$Ln(CS_t) = \ln(k) + m \times Ln\left(x \times CI_t + (1-x) \times CQ_t\right) \tag{6}$$

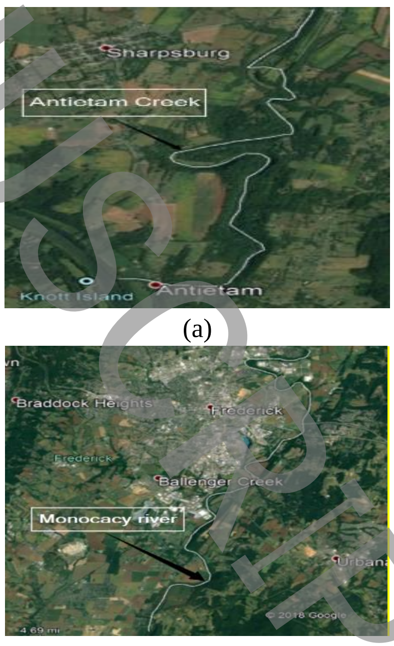
It can be said that by knowing the weight coefficient x and creating a linear regression between parameters $\mathrm{Ln}(\mathrm{CS_t})$ and $\mathrm{Ln}(\mathrm{x} \times \mathrm{CI_t} + (1-\mathrm{x}) \times \mathrm{CQ_t})$, the slope of the line is equal to m and the point of intersection with the vertical axis will be equal to $\ln(k)$ .Also, Eq. 7 is used to calculate the weighting factor x.

$$x = \frac{1}{2} - \frac{Q_0}{2S_0 Bc\Delta X} \tag{7}$$

Where Q_0 is river discharge, S_0 is the longitudinal slope, B is average reach width, c is celerity velocity which is calculated from c = 1.67U$_0$ so that U$_0$ is equal to the average velocity, and ΔX is assumed as reach length.

2.2. Data Series

The concentration-time data series which is used in this study were collected by the USGS through Monocacy river and Antietam creek in the United States. During the tests to collect concentration-time data, rhodamine contaminant was used as a mass conservative tracer, the pollution was injected suddenly through the river. It should be noted that in Muskingum's linear and non-linear routing methods, the mass conservation should be satisfied. Therefore, it can be said that the operated model is consistent with the nature of the transfer problem. The number of data collection locations was 4, and (4-8) for Antietam Creek and Monocacy River, respectively. Fig. 1 shows satellite images of the studied area from both rivers.



(a)

(b)

Fig. 1, (a) Satellite image from Antietam Creek, (b) Satellite image from Monocacy River

3. Results and discussion

Fig. 2 shows an example of established fits. By continuing the method and extracting two model parameters of k and x, sometimes it was found that the reconstructed curves do not have the necessary accuracy, so using the least-squares optimization technique and based on pollutant input and output data, Eq. 4 was used for the desired reach length, and new coefficients were extracted. The results show that both methods have good performance for deriving the parameters of Muskingum nonlinear model and the extracted triple pairs are sometimes not within the typical limits defined for the linear model.

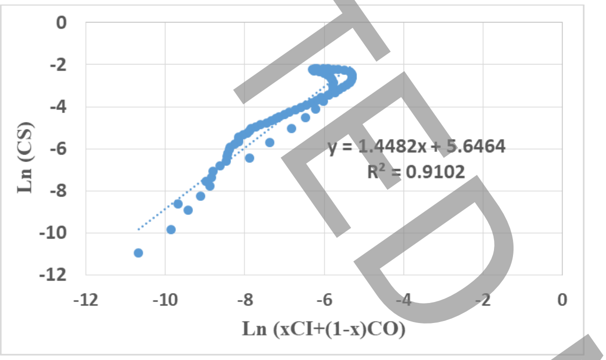


Fig. 2, The regression line establishment between the pollutant storage and the nonlinear Muskingum storage parameter

Fig. 3 gives several acquired concentration-time curves versus simulated curves using the nonlinear Muskingum method at four locations of the Monocacy River. It can be seen that the nonlinear Muskingum method can satisfactorily model the concentration-time curves by applying a time shift corresponding to the average flow velocity through the river reach. Additionally, the accuracy of the extracted parameters and the extent and manner of their variation can also include valuable information. Therefore, the extracted coefficients were statistically analyzed after classification and normal probability curves were fitted over them.

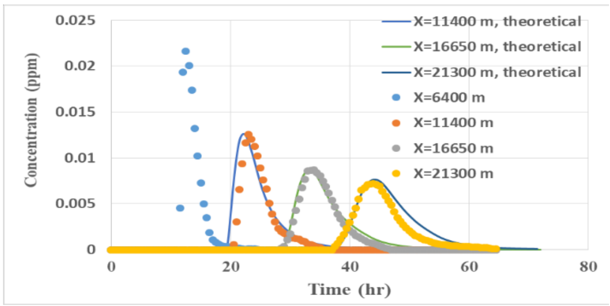


Fig. 3, Simulation of concentration-time curves in Monocacy River with the discharge of 5.9 m^3 /s

4. Conclusion

In this study, the nonlinear Muskingum hydrological method of flood routing was used to simulate the concentration-time curves of pollutants through the river reaches. The results show that due to the similarity of the shape of the pollutant breakthrough curves to the inflow and outflow flood hydrographs, this model can reconstruct the dimensions of the breakthrough curves by extracting its triple coefficients. However, the simulation faced some problems regarding the transfer phenomenon by average velocity. It was found that the extracted curve should be transferred as (L / u) on the time axis to achieve an exact match between the observed and computational concentration-time curves. The triple sets of calculated model parameters showed that, contrary to what exists in flood routing, the range of these parameters is somewhat different in the pollution transport problems.

5. References

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