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Application of MQ-RBF method for solving seepage problems with a new algorithm for optimization of the shape parameter

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ABSTRACT: The accuracy of the meshless method, Multiquadric, depends completely on the choice of its optimal shape parameter. The purpose of this research is proposing a new algorithm for determining the optimal shape parameter. It resolves some of the previous difficulties, such as depending on the number of computational nodes or an exact solution of the problem, high cost and low accuracy of calculations, being experimental, convergence of classical optimization methods to local optimal points and so on. For this purpose, in addition to introducing a new objective function, Genetic Algorithm(GA) was used and for speeding up the process of its solution, lower bound and upper bound of the shape parameter are suggested as minimum (when the coefficient matrix is not singular) and maximum radius of computational nodes, respectively. The algorithm consists of four steps: 1) producing initial shape parameters by GA in the proposed range, 2) introducing the MQ function with a few numbers of computational points, 3) introducing the MQ function with a large number of computational points, and 4) minimizing the difference between solutions of two functions obtained from the two preceding steps. In the meta-heuristic algorithm, uniform and non-uniform regular distributions of computational nodes have been successfully applied and it was shown that with this approach, an optimal constant shape parameter independent of the number of computational points could be obtained for arbitrary geometries. For verification, examples of homogeneous, inhomogeneous and anisotropic types of the seepage phenomena were solved so that domain decomposition technique was used for inhomogeneous problems and complex geometries. A comparison of results with other exact and numerical solutions showed the high ability and accuracy of the proposed algorithm.

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1. INTRODUCTION

Many researchers have successfully applied the Multiqudric meshless method which is probably the most attractive member of the radial base functions family for solving partial differential equations [1-5]. The accuracy of this method depends strongly on its shape parameter. So far, researchers have been working on the method to be developed and optimized [6-11]. Previous approaches suffer from deficiencies such as 1) being experimental, 2) dependence of the shape parameter on the number of computational nodes, 3) unknown lower and upper limits of the shape parameter, 4) convergence of classical optimization methods to local optimal points and so on. The purpose of the present study is to propose a new adaptive algorithm that solves some of the previous problems for determining the optimal shape parameter. In this approach, the genetic algorithm is used with a new objective function and to accelerate the solution process, lower and upper limits for the optimal shape parameter are proposed. The algorithm is also tested for solving homogeneous, inhomogeneous, and anisotropic seepage problems.

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2. METHODOLOGY

The governing PDE for the 2D inhomogeneous and the anisotropic problem of seepage in a steady state is the Laplace equation as follows:

$$K_{xx}\frac{\partial^2 h}{\partial x^2}+K_{yy}\frac{\partial^2 h}{\partial y^2}=0 \tag{1}$$

Where h is the total head and k_{xx} k_{yy} are the permeability coefficients in the principal directions. MQ approximates solution of 2D differential equations with the following estimation function and obtains its values at any point in the computational domain:

$$f(x,y)=\sum_{j=1}^{N}\lambda_j\sqrt{1+((x-x_j)^2+(y-y_j)^2))/c^2} \tag{2}$$

in which (x_j, y_j) are components of the computational points and λ_j are unknown coefficients which will be obtained using N points in the computational domain. Also, c is the shape parameter. In this study, to select the optimal value of c, a new algorithm that eliminates some of the weaknesses

Table 1. Results of Example 1 with the triangular distribution of nodes

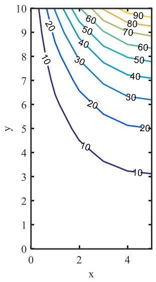
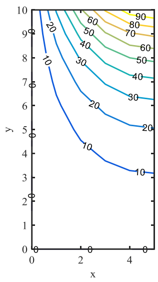
N	c	Head (Exact)	Head (MQ)
55	0.25		
80	0.25		
117	0.25		
148	0.25		
179	0.25		

Table 2. Total heads in an inhomogeneous earth dam

x	y	H (FV)	H (MQ)
0.250	0.500	3.000	3.000
1.007	0.500	2.900	2.910
0.437	0.500	2.980	2.980
1.160	0.500	2.700	2.700
0.606	0.500	2.960	2.970
1.315	0.500	2.500	2.500
0.763	0.500	2.940	2.950
1.477	0.500	2.300	2.300
0.917	0.500	2.920	2.930
1.653	0.500	2.100	2.100

of the other researches is proposed in the following steps:

1- Generating the initial shape parameter by GA in the $[r_{min}, r_{max}]$ where:

$$r_{\min} = \min\left(\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}\right), \quad i, j = 1, 2, \ldots, N, \quad i \neq j \tag{3}$$

$$r_{\max} = \max\left(\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}\right), \quad i, j = 1, 2, \ldots, N, \quad i \neq j \tag{4}$$

2- Creating an MQ function with a minimum number of computational points ($f_1(x, y)$).

3- Creating an MQ function with the maximum number of computational points ($f_2(x, y)$).

4- Defining the objective function as:

$$Fitness = \sum_{i=1}^{m} \frac{\left|f_1(x_i, y_i) - f_2(x_i, y_i)\right|}{\left|f_1(x_i, y_i)\right|} \tag{5}$$

Therefore GA introduces the optimal shape parameter which minimizes the objective function.

3. RESULTS AND DISCUSSION

Several examples are presented using the proposed algorithm. In the first example, the seepage is investigated in a

Table 3. Velocity values in an anisotropic earth dam

x	y	V (FV)	V (MQ)
0.2501	0.5002	0.000789525	0.00077029
0.5182	0.5002	0.001259283	0.001228734
0.6466	0.5002	0.000883665	0.000857811
0.7645	0.5002	0.000850473	0.00083186
0.8819	0.5002	0.000848906	0.000825534
1.0001	0.5002	0.000849079	0.000821334
1.1179	0.5002	0.000848906	0.000818733
1.2358	0.5002	0.000850473	0.000828244
1.3536	0.5002	0.000883665	0.000856491
1.4814	0.5002	0.001259283	0.001228734

homogeneous and isotropic domain and other examples will be expressed in inhomogeneous (with domain decomposition technique) and anisotropic earth dams.

Results show that the optimal shape parameter does not depend on the number of computational points. This result has been achieved in all other examples.

4. CONCLUSION

For the first time, using the presented approach, the optimal and constant shape parameter in the MQ method has been obtained for any number of points. In other words, the shape parameter does not depend on the number of computational points and unlike other methods, it is not necessary neither to be optimized for any number of computational nodes nor a lot of computational time. Also, for speeding up the process of solution, lower and upper limits of the shape parameter have been successfully suggested as a minimum (when the coefficient matrix is not singular) and maximum of radius of computational nodes. For the numerical approach to be evaluated, homogeneous, inhomogeneous and anisotropic problems of seepage phenomena with application in the body and foundation of earth dam were investigated. The results showed high capability and accuracy of the proposed MQ algorithm compared to the analytical solution and the finite volume method results.

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