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One-Dimensional Simulation of Water Hammer in Non-Newtonian Fluids

A. Khamoshi [1], A. Keramat [1,*], A. Majd [2]

[1] Civil Engineering Department, Jundi-Shapur University of Technology, Dezful, Iran

[2] Khuzestan Water and Power Authority (KWPA), Ahwaz, Iran

ABSTRACT: Unlike previous studies in Non-Newtonian fluids that use complex two-dimensional models to calculate the velocity gradient in this research, one-dimensional models have been used to calculate Non-Newtonian losses that can be implemented faster and have higher execution speeds. The main objective of this research is to study the phenomenon of water hammer in Non-Newtonian fluids of power type (Power Law) using Brunon and Zeilke models. In order to calculate the shear stress in relation to the momentum of the Zeilke and Brunon model, and to solve the equations, the line characteristics of the nonlinear fluid solution have been used. The Brunon model is based on the assumption that the shear stress of the wall changes due to the acceleration of the acceleration, proportional to the acceleration of the fluid. Zilck's method for calculating the unsteady friction coefficient presents a model based on the analytic integral of convolution. The velocity gradient in the steady state is used to obtain the velocity gradient in the Zeilke model. Finally, numerical results are compared with the results of another research to ensure the accuracy of the solution algorithm. The results of Non-Newtonian fluid modeling show significant changes in pressure values. The proposed formulas, similar to the two-dimensional models, can simulate these changes. As expected in the same continuous flow conditions, the maximum pressure decreases with decreasing viscosity of the fluid. In other words, by decreasing the viscosity of the fluid, the amount of drops across the pipe path will be reduced. According to expectations in the steady flow conditions, the maximum error in the maximum pressure at the valve location is about one percent higher than the two-dimensional state, which, with a decrease in the viscosity of the fluid, causes this error to be close to zero.

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1. INTRODUCTION

Pressure surges are commonly encountered in both natural and engineering systems, such as oil transportation and human arterial networks. Precise simulation of these transients needs to elaborate on unsteady friction modeling. The most prominent research in the field of unsteady friction in Newtonian fluids is Zeilke's analytical article [1] where he obtained analytical relations to handle this phenomenon. Brunone et al. [2] adopted an additional term including a K factor to simulate the local momentum. The factor is effectively calibrated by Pezzinga [3]. Vardy and Brown [4] have been performed significant contributions to non-Newtonian unsteady pipe-flows especially modeling fluids with time-dependent viscosities. More recently, Wahba [5] compared shear-thinning and shear-ticking fluids in response to a fluid hammer event using the power-law model. Subsequently, Majd et al. [6] investigated power-law and Cross fluids.

2. GOVERNING EQUATIONS

2.1. Transient flow

The momentum and continuity equations in the following form are used in this research [7].

*Corresponding author's email: keramat.alireza@gmail.com

$$\frac{\partial H}{\partial z}+\frac{1}{gA}\frac{\partial Q}{\partial t}+\tau\frac{Q|Q|}{2DgA^2}=0 \quad , \tau=\tau_s+\tau_u \tag{1}$$

$$\frac{\partial H}{\partial t}+\frac{a^2}{gA}\frac{\partial Q}{\partial z}=0 \tag{2}$$

where Q is volumetric flowrate, H is the piezometric head, t is time, z is the distance along the pipe centerline, g is gravitational acceleration, D is inside pipe diameter, A is a cross-sectional flow area, a is wave speed of the fluid and τ is shear stress at pipe wall comprised from unsteady component τ_u (to be quantified later) and steady component τ_s:

$$\boldsymbol{\tau_s}=\boldsymbol{\lambda}\frac{\boldsymbol{L}}{\boldsymbol{D}}\frac{\boldsymbol{V}^2}{\boldsymbol{2g}} \tag{3}$$

in which, $\boldsymbol{L}$ is pipe length, $\boldsymbol{\lambda}$ is Darcy–Weisbach friction factor to be computed for laminar flows as:

$$\boldsymbol{\lambda}=\frac{\mathbf{Re}}{\mathbf{64}} \tag{4}$$

in which, $\boldsymbol{Re}$ Reynolds number. Eq.s (3) and (4) are valid for Newtonian fluids. Chhabra and Richardson [8] derived $\boldsymbol{\tau_s}$ the relation for non-Newtonian fluids:

$$\tau_s = m\left(\frac{8V}{D}\left(\frac{3n+1}{4n}\right)\right)^n \tag{5}$$

in which m and n are constants of the power-law fluid.

2.2. Unsteady friction in Newtonian fluids

2.2.1. Zeilke model

The unsteady component τ_u in Eq. (1) is found by:

$$\tau_u = \frac{32\vartheta A}{DQ|Q|}\int_0^t \frac{\partial Q}{\partial t^*} W\left(t-t^*\right)dt^* \tag{6}$$

in which ϑ is kinematic viscosity. Assuming linear unsteady friction, Zeilke [1] derived this weight function

$$W\left(\tau^*\right) = \frac{A^* e^{-\frac{\tau^*}{C^*}}}{\sqrt{\tau^*}}\ ,\ \tau^* = 4\vartheta t / D^2 \tag{7}$$

where A^* and C^* are constants in laminar flows [9].

2.2.2. Brunsone's model

In this model τ_u depends on the time and spatial derivative of fluid velocity V as well as flow direction $\text{sign}(V)$ [2]:

$$\tau_u = \frac{KD}{V|V|}\left[\frac{\partial V}{\partial t} + a\,\text{sign}(V)\left|\frac{\partial V}{\partial x}\right|\right] \tag{8}$$

$$K = \frac{\sqrt{C^*}}{2}\ .\ C^* = \begin{cases} 0.00476 & Re \le 2300 \\ \dfrac{7.41}{\text{Re}^{\log\left(14.3\text{Re}^{-0.05}\right)}} & Re > 2300 \end{cases} \tag{9}$$

2.3. Non-Newtonian power-law fluid

Among the wide range of non-Newtonian liquids, a type of time-independent ones namely power-law fluids are considered [8]. In this type of fluids, the fluid dynamic viscosity $\boldsymbol{\varsigma}$ is related to the velocity gradient $\boldsymbol{dV_z / dr}$ by means of two specified coefficients m and n:

$$\boldsymbol{\eta = m(\tfrac{dV_z}{dr})^{n-1} = m(\dot{\gamma})^{n-1}} \tag{10}$$

where r stands for radial direction and $\boldsymbol{V_z = V_z(r)}$ denotes the cross-sectional velocity profile.

2.4. Unsteady friction in non-Newtonian fluids

2.4.1. Brunsone's model

To implement this model, the coefficient K in Eq. (8) is fundamental. To account for non-Newtonian fluids in this equation, Reynolds number (Re) has to be accordingly defined [10]:

$$\mathbf{Re} = \frac{8\rho V^{2-n} D^n}{m(6+2/n)^n} \tag{11}$$

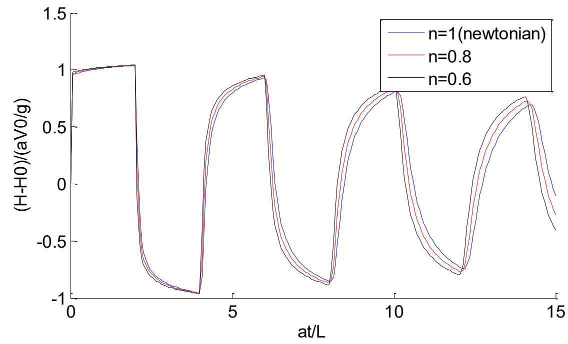

Fig. 1. Head at the valve located in the present study based on Zeilke's model

Using Eq. (11), C^* in Eq. (9) is found in Brunone's model:

$$C^* = 0.015\left(\frac{0.28\rho V^{1-n} D^n}{m(6+2/n)^n}\sqrt{\frac{m(\frac{8V}{D}\left(\frac{3n+1}{4n}\right))^n}{\rho}}\right) \tag{12}$$

2.4.2. Zeilke model

Considering Eq. (7), the weight function in Zeilke's model is computed according to τ^* which itself depends on the kinematic viscosity $\vartheta = \eta / \rho$ which is made available using Eq. (10). Furthermore, a close investigation of power-law fluids during steady-state allows for computing the velocity gradient at the pipe wall [8]:

$$(\frac{dV_z}{dr})_w = \left[\frac{3n+1}{4n}\right]\left(\frac{8V}{D}\right) \tag{13}$$

Eq. (13) in combination with Eq. (10) provides τ^*:

$$\tau^* = \frac{4\vartheta t}{D^2} = \frac{4\left(\frac{m(\frac{dV}{dr})^{n-1}}{\rho}\right)t}{D^2} = \frac{4\left(\frac{m(\left[\frac{3n+1}{4n}\right]\left(\frac{8V}{D}\right))^{n-1}}{\rho}\right)t}{D^2} \tag{14}$$

3. VERIFICATION OF THE PROPOSED MODEL

The numerical results of the proposed solution (h^{1D}) were compared with those of Majd et al. (h^{2D}) [6] for a copper pipe, D = 0.025m, L = 36.09 m, Re = 82 and a = 1324 m/s contained by high-viscosity oil of μ=0.03484 N.s/m². Figure 1 depicts computed heads at the valve (Zeilke model) and Figure 2 displays error (Eq. (15)) between the two aforementioned models versus time computed by:

$$\text{error}(t) = h^{2D}(t) - h^{1D}(t) \tag{15}$$

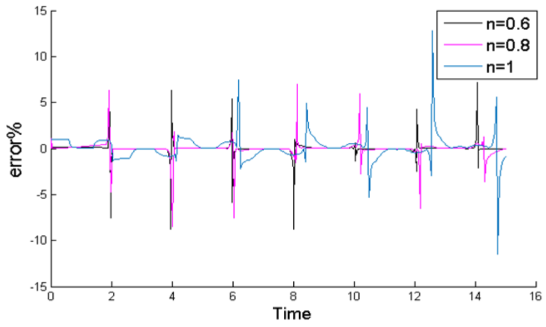

Fig. 2. Error propagation of the present model with respect to Majd et al.'s [6] study.

4. CONCLUSIONS

A fast, efficient and simple approximation for transients in non-Newtonian fluids was developed. The method was based on the well-known unsteady friction models of Brunone and Zeilke which was herein modified to treat power-law fluids during fluid hammer. The core idea behind this study was the computation of the shear stress only at the pipe wall instead of the whole flow cross-section. This stress was computed by the existing relation between the non-Newtonian viscosity of the fluid and the wall-shear stress which was derived for steady-state conditions but herein was exploited temporarily during transients. The proposed model was validated against verified and accurate two-dimensional transient results from literature and reveal acceptable approximation to fluid hammer simulation in power-law fluids.

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