

An Investigation on the Capacity of Membrane Action of Restrained Slender Reinforced Concrete Beams

Mehdi Koohestani¹, Mohammad Reza Esfahani^{2*}

¹ M.Sc. Structural Engineering, Department of Civil Engineering, Faculty of Engineering, Ferdowsi University of Mashhad

² Professor, Department of Civil Engineering, Faculty of Engineering, Ferdowsi University of Mashhad

ABSTRACT

In recent years, several studies have been conducted on membrane action of restrained reinforced concrete beams. Several factors and parameters are involved in the analysis and calculation of this phenomenon. Determination of the effect of each parameter on the response of the load-carrying capacity of these beams is the main purpose of this study. An analytical model for the analysis of slender restrained reinforced concrete beams with different span-depth ratios is used to investigate the effect of four parameters of concrete compressive strength, reinforcement ratio, axial stiffness and rotational stiffness of the support. This analytical model is formulated based on a sectional analysis approach to establish equilibrium and compatibility conditions. Programming of the model is carried out with FORTRAN software.

The results show that with increasing the span-depth ratio, the effect of membrane action on the capacity of beams decreases. It is also observed that the effect of all four parameters on the load-carrying capacity is greater in short beams. An increase of 0.75% in the concrete compressive strength with a span-depth ratio of 9, the load-carrying capacity increases to 111 kN, while for the ratio of 31, this value increases only to 26 kN. The load-carrying capacity response is nonlinear with the amount of support stiffnesses. In evaluating the effect of support stiffnesses on the membrane action response, it was observed that membrane action capacity increases with restraint stiffness only in the regime of weak restraints.

KEYWORDS

Membrane action, Beam, Slender, Reinforced concrete, Analytical model

* Corresponding Author: Email: esfahani@um.ac.ir

1. Introduction

Recently, resistance mechanisms of reinforced concrete (RC) buildings against progressive collapse have been investigated, extensively. One of the favorable structural mechanism to mitigate the progressive collapse in RC structures and increase the load-carrying capacity is the mechanism of compressive membrane action. As it is shown in Fig. 1, restraining the beam between supports prevents the increase in the length of the beam. This causes an axial compression force along the beam, which is transmitted to the supports in an arc-shaped path through the middle of the beam.

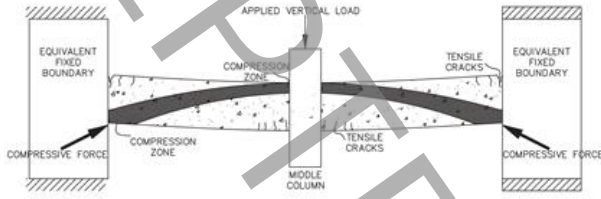


Figure 1. Schematic of compressive arch action in RC beam [1].

Various efforts have been made to introduce effective methods for calculating and considering the membrane force and using it in structural design. Yu and Tan [2] took an important step in this regard by presenting a numerical method to calculate the compressive axial force. These numerical methods provide a better understanding of the mechanics of lateral limitation effect on the capacity and response of structural members. Another model presented in this area is by Wu [3], based on equilibrium and compatibility conditions along the beam for predicting the nonlinear response of RC beams. Mansour [4] by completing this model and considering the second-order effects in numerical modeling, takes an effective step towards improving the proposed method for analyzing RC beams with axial and rotational restraints at supports. In this research, using the numerical method provided by Mansour [4], the study is conducted on restrained slender RC beams.

2. Methodology

The proposed model is formulated using a sectional analysis approach that assumes plane sections remain plane, and accounts for both material nonlinearities and second-order effects.

A fully rigid support condition can be defined using the following compatibility conditions. No lateral expansion and rotation of the beam at supports is allowed.

$$L_{c,ce} = L_0 \quad (1)$$

$$\int_0^L \varepsilon_t = \int_0^L \varepsilon_b \quad (2)$$

where L_0 is the initial undeformed length of a beam, $L_{c,ce}$ is the chord length of the centroidal axis, ε_t is the strain along the top of a beam, and ε_b is the strain along the bottom of a beam. The calculation of the $L_{c,ce}$ is shown in the following equation:

$$L_{c,ce} = \frac{L_0}{L_w} L_{a,ce} \quad (3)$$

where L_w is the arc length of the deflected shape, $L_{a,ce}$ is the arc length of the centroidal axis. The L_w and $L_{a,ce}$ can be determined using the following equations,

$$L_w = \sum_{i=2}^n \sqrt{(w_i(i) - w_i(i-1))^2 + (x(i) - x(i-1))^2} \quad (4)$$

$$L_{a,ce} = L_0 + \sum_{i=2}^n \left(\frac{\varepsilon_{ce}(i) + \varepsilon_{ce}(i-1)}{2} \right) (x(i) - x(i-1)) \quad (5)$$

where n is the total number of sections in the beam, and x is the horizontal distance between section i and the support. $\varepsilon_{ce}(i)$ and $w_i(i)$ are concrete strain at the centroid and resultant deflection at the section i , respectively. In the following, the rotation of a beam at the support can be calculated as:

$$\theta_0 = \sum_{i=1}^{n-1} \left(\frac{\phi(i) + \phi(i+1)}{2} \right) (x(i+1) - x(i)) \quad (6)$$

where $\phi(i)$ is the curvature at a section i .

The proposed model operates by an iteration procedure and is based on two layered and sectional analysis. As shown in Fig.2, iterations are carried out in three levels.

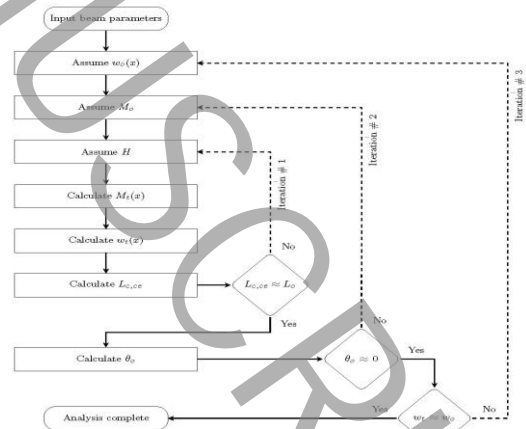


Figure 2. Formulation for the analytical model Proposed by Mansour [4].

3. Discussion and Results

Five beams with the same geometrical properties but different span to depth ratios of $L/h = 9, 18, 22, 27, 31$ ($h = 225\text{mm}$ constant) are considered to evaluate the effect of each parameter on the membrane action in restrained slender RC beams.

After comparing the results for each of the parameter of concrete compressive strength, reinforcement ratio, axial stiffness and rotational stiffness of the support and the span-depth ratio on the load-carrying capacity Q , lateral restraint load H and maximum midspan deflection response w of the beams, the effects of each parameter is shown in Fig. 3

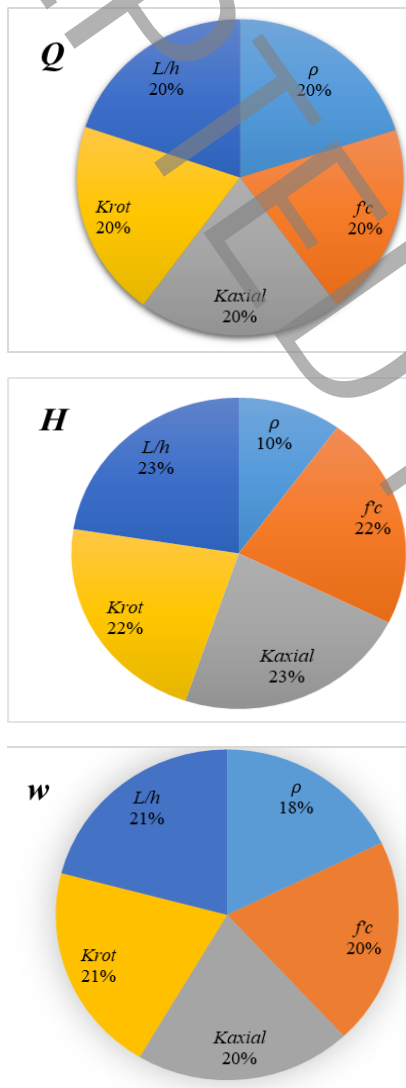


Fig. 3. The effect of different parameters on the responses of load-carrying capacity, lateral restraint load and maximum midspan deflection of the restrained reinforced concrete beams.

As seen in Fig.3, all five parameters have similar effect on the response of the load-carrying capacity. The compressive axial force response, the span-depth ratio and axial stiffness of the support have the greatest impact. Finally, for the maximum midspan deflection response, the span-depth ratio and rotational stiffness of the support have the most effect.

4. Conclusions

The main conclusions can be summarized as follows:

With increasing the span-depth ratio, the effect of membrane action on beams decreases. It is also observed that the effect of concrete compressive strength, reinforcement ratio, axial stiffness and rotational stiffness of the support on load-carrying capacity is greater in short beams. The load-carrying capacity response is nonlinear with the amount of support stiffnesses. In evaluating the effect of support stiffnesses on the membrane action response, it was observed that the membrane action capacity increases with restraint stiffness only in the regime of weak restraints.

5. References

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