A new method for determining natural modes and their frequencies with the concept of node in vibrations of M-DOFs

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ABSTRACT

This paper evaluates the vibration of M-DOF systems by calculating the natural frequencies and mode shapes. The introduced method is established on the base of node concept, which is the point of a mode shape with zero displacement. In this method a system with two or more degrees of freedom is transformed to two or more isolated systems with one- DOF. Those systems are isolated in node places and vibrate with same frequencies in every mode. Each spring which located between two adjacent lumped mass will be converted to series combination of two separated springs. The stiffness of the first spring is equal to the effective stiffness of the two series separated springs. Proposed method provides a good physical understanding about the concept of vibration modes. Besides, this method is accurate and sometimes is simpler and quicker than common method.

KEY WORDS: Natural modes, natural frequencies, node, M-DOFs system**,** one- DOF systems

1. Introduction

Vibration modes and their frequencies are important subjects in dynamic of structures and the relevant references [1]. Namjooyan *et al*. [2] obtained a new relation for empirical period time of moment frames and evaluated the accuracy of the offered equations in Iranian Standard No. 2800 [3]. Ahmadi Danesh and Rafiee [4] evaluated the upper modes effects on the behavior of tall buildings. The studied buildings were 5, 12, 18, 30 and 50 story buildings. They realized that the upper modes are more effective on the upper and middle stories of fairly tall (12&18 story) and very tall (30&50 story) buildings, respectively. Do Hyun Kim and Ji Young Kim [5] by field measurement and FEM determined the natural frequencies of three first modes of multy-story reinforced concrete buildings with various structural systems. They observed that the value of concrete elasticity modulus, existence of non-structural members and flexural stiffness of slabs have effects on dynamic parameters. Modified Energy Method was employed for dynamic analysis of SDOF by Jalili Sadrabad et al. [6]. In this method potential, dissipation and kinematic energy were defined in accordance with spring, dissipation and inertia forces. This approach was done by integrating of two sides of motion equation of SDOF, accordingly "dx" was written as "dx=v(t)dt". In this way, energy equations were converted into definite integrals in time domain.

In the literature review [1], [7-9] the mass-spring model of a shear-building structure must firstly be drawn for its vibration evaluating. Lumped masses and springs are equivalent of stories masses and shear stiffness between stories, respectively. By making mass and stiffness matrices and doing some mathematical operations, characteristic matrix and characteristic equation could be obtained. Frequencies of modes are the roots of the characteristic equation.

In almost all the researches, frequencies and relevant shape modes were calculated by *Eigenvalues Problem* and *characteristic equation*. Here, this problem is solved by a new and simple method and on the base of physical primary concepts of mass, spring and node of vibration. To enter the main text, present of some concepts is suitable:

Rigid mode: One of the vibration modes of unstable structures in which all the masses have similar movements.

Series and Parallel springs: Two or more springs are said to be in series when they are connected end-to-end or point to point and it is said to be in parallel when they are connected side-by-side.

Node: In mode shapes, point(s) of system which has (have) no movement is (are) node(s) of vibration.

2. Methodology

2.1 Shape modes and frequencies in 2-DOF Unstable systems

This system has no support, thus is unstable and has rigid mode (Fig. 1).

2.1.1 Rigid mode

In this mode the spring has no deformation, consequently its mode shape is $\phi_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$ and the relevant frequency is equal to zero, $\omega_1 = 0$.

2.1.2 non-rigid mode

According to the node concept, a point is considered between two masses. In this mode, the movements of masses are against of each other, therefore the shape mode can be written as $\phi_2 = \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix}$.

Fig.1: Dividing the primary spring (left) into two adjust series springs (K_1&K_2) from the node location (right)

A support can be imagined in the location of the node, accordingly each of the masses is connected to a rigid support via separated springs, "K_1" and "K_2" (Fig.1). Thus the frequency of the second mode will be equal to:

$$\omega_2 = \sqrt{\frac{K_1}{m_1}} = \sqrt{\frac{K_2}{m_2}} \tag{1}$$

Besides, the first spring (K) is the equivalent spring of series springs, K_1 and K_2, i.e. $K = \frac{K_1 K_2}{K_1 + K_2}$.

Therefore K_1, K_2 and the second mode frequency can be calculated. In every moments of the free vibration, the axial forces in the separated springs are the same ($F_1 = F_2 = |K_1\phi_{12}| = |K_2\phi_{22}|$). By regard to Eq. 1, the second shape mode, modal matrix and non-rigid (second) mode frequency will be obtained:

$$\frac{\phi_{12}}{\phi_{22}} = -\frac{K_2}{K_1} = -\frac{m_2}{m_1} \quad \Rightarrow \phi_2 = \begin{Bmatrix} 1 \\ \frac{\phi_{22}}{\phi_{12}} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -\frac{m_1}{m_2} \end{Bmatrix} \quad , \Phi = \begin{bmatrix} 1 & 1 \\ 1 & -\frac{m_1}{m_2} \end{bmatrix} \quad , \omega_2 = \sqrt{\frac{K_1}{m_1}} = \sqrt{K\frac{m_1 + m_2}{m_1 m_2}} \tag{2}$$

2.2 Shape modes and frequencies in 2-DOF Unstable systems

2.2.1 Stable systems which have a support

Fig.2: A 2-DOF stable system which has a support (left), assuming node location and dividing the primary spring into K_{21} and K_{22} (right)

This system is not unstable, thus rigid mode will not form. A suitable location must be assumed as a node to determine the shape modes and corresponding frequencies. Between the masses is that location (Fig.2). According to Fig. 2 the primary spring is the equivalent spring of the two new springs, i.e. $K_2 = \frac{K_{21} K_{22}}{K_{21} + K_{22}}$. The frequencies of the two modes satisfies the next equation:

$$\omega_i = \sqrt{\frac{K_{e1}}{m_1}} = \sqrt{\frac{K_{e2}}{m_2}} \qquad \text{where} \qquad K_{e1} = K_1 + K_{21} \qquad K_{e2} = K_{22} \tag{3}$$

By solving simultaneously the last equations:

$$K_{21}{}^2 + (K_1 - K_2 - \frac{m_1}{m_2} K_2) K_{21} - K_1 K_2 = 0 \tag{4}$$

Top equation has two roots for "K_{21}", each of them must be used for a mode. Then the value of "K_{22}" will be obtained. Therefore shape modes and frequencies can be determined. Other systems with upper DOFs are evaluated in the main text.

3. Results and Discussion

Example:

The frequencies of the second mode of unstable (Fig.1) and stable systems (Fig.2) with the below characteristics are requested: unstable system: m_1=1kg, m_2=2kg, K= 1000N/m. stable system: m_1=1kg, m_2=2kg, K_1=1000N/m, K_2=2000N/m.

Solution with the proposed method:

Unstable system: $$\omega_2 = \sqrt{\frac{K_1}{m_1}} = \sqrt{K\frac{m_1+m_2}{m_1 m_2}} = \sqrt{1000\times\frac{1+2}{1\times 2}} = 10\sqrt{15} \approx 38.7\,rad/s$$

Stable system: Eq. (4): $$K_{21}{}^2 - 2000K_{21} - 2\,000\,000 = 0 \Rightarrow K_{21} = 1000(1\pm\sqrt{3}),$$

Eq. (3): $$\omega_i = \sqrt{\frac{K_{e1}}{m_1}} = \ldots = \sqrt{\frac{K_1+K_{21}}{m_1}} \Rightarrow \omega_2 = \sqrt{\frac{1000+1000(1+\sqrt{3})}{1}} \approx 61.1\,rad/s$$

The solution process shows that the application of the introduced method is very simple and fast and the results are exact.

4. Conclusion

In this paper frequencies and mode shapes of dynamic systems are calculated on the base of node concept in vibration. Node is a point of a mode shape with zero displacement. Application of the proposed method is very simple and does not need complex physical and mathematical operations. The method is not approximate, thus the results are exact.

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