A fuzzy approach for designing of subway lines, case study: development of the Tehran subway network

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Abstract

Subway network design can be classified as one of the most challenging problems in transportation planning, where different deterministic or non-deterministic approaches have been utilized for optimal design. Non-deterministic methods, having fewer limitations and representing reality with its intrinsic uncertainty, have thus been the focus of less research. This paper incorporates concepts of fuzzy set theory into optimal design of subway networks to the case of Tehran. Two binary mathematical programming models with different objective functions are developed. The first model maximizes the covered population while minimizing construction cost, whereas the second maximizes the ratio of the covered population to construction cost. These objective functions are modeled in both a fuzzy and a deterministic state. In the fuzzy model, we use a fuzzy penalty factor instead of edge length constraints and propose a Sugeno fuzzy inference system for calculating the covered population. Results indicate that the total length of designed lines with the linear and nonlinear fuzzy approach is equal to 139.3 km (477000 billion Iranian Rials) and 144.6 km (494000 billion Iranian Rials), respectively. Considering topology improvement per construction cost index, designed lines with the linear fuzzy model are better than the nonlinear fuzzy model. In comparison to the classic deterministic approach, the proposed fuzzy approach can improve topology improvement per construction cost index by 23 percent.

Keywords

Fuzzy mathematical programming, Fuzzy factors, Subway line design, Non-deterministic approach, Line locating.

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1. Introduction

The Subway network is known as an essential and important transit mode in huge cities. Subway networks projects are major endeavours that require long-term planning. This paper proposes a fuzzy approach for designing of subway lines.

2. Methodology

This paper compares four mathematical programming models, described in the following, for designing of subway lines:

1. A linear mathematical programming (LP)
2. A non-linear mathematical programming (NLP)
3. A linear mathematical programming including the proposed fuzzy penalty coefficient of edge length and the proposed fuzzy approach for population coverage calculating (FLP)
4. A non-linear mathematical programming including the proposed fuzzy penalty coefficient of edge length and the proposed fuzzy approach for population coverage calculating (FNLP)

Formally, the metro network design problem is that of embedding a set of interconnected transit lines within a larger undirected network G=(N,E), where N={1,.,n} is a node set and E={(i,j): i,j∈N, i < j} is an edge set. The nodes correspond to population centroids in a city, while the edges correspond to potential connections to be built between vertex pairs. the objective is a linear combination of the construction cost and of the population covered by the network. Let xij be a binary variable equal to 1 if and only if edge (i,j) belongs to the network, let yi be a binary variable equal to 1 if and only if vertex i belongs to the network, and let a be a user-controlled positive parameter. Let T ⊂ N be a set of nodes that must necessarily belong to the network (T may be empty) and d_{ij} is a fuzzy penalty factor utilized instead of edge length constraint.

$$\min Z = \sum_i c_i y_i + \sum_{i,j} L_{ij} c_{ij} d_{ij} x_{ij} - \sum_i P_i y_i \quad (1)$$

$$\max Z = \sum_i P_i y_i \Big/ \left(\sum_i C_i y_i + \sum_{ij} L_{ij} c_{ij} d_{ij} x_{ij} \right) \quad (2)$$

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$$\max Z = \sum_i P_i y_i \Big/ \left(\sum_i C_i y_i + \sum_{ij} L_{ij} c_{ij} x_{ij} \right) \quad (4)$$

$$\sum_{i,j} x_{ij} = \sum_i y_i - 1 \quad (4)$$

$$\sum_{i,j \in S} x_{ij} \le \sum_{i \in S/\{k\}} y_i, k \in S \subseteq N, |S| \ge 2 \quad (5)$$

$$\sum_{i \in V/\{i\}} x_{ij} = y_i \quad (6)$$

$$\sum_{j \in V/\{j\}} x_{ij} = y_j \quad (7)$$

$$\sum_{i,j \in S} x_{ij} \ge y_k, k \in S \subseteq N, |S| \ge 2 \quad (8)$$

$$500 \le L_{ij} \le 2000; \forall ij \in E \quad (9)$$

$$y_i = 0,1; i \in N \quad (10)$$

$$x_{ij} = 0,1; i, j \in N \quad (11)$$

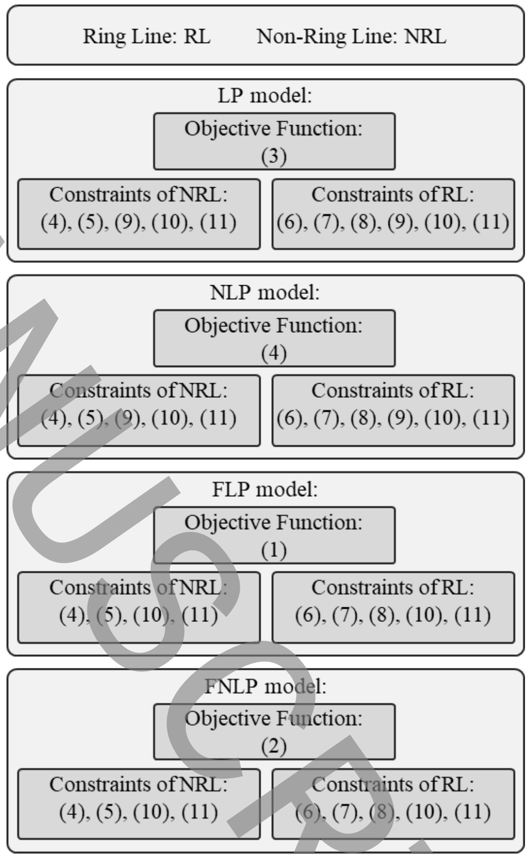

Figure 1. Utilized mathematical programming models

Figure 1 depicts utilized models for designing of subway lines. Designed networks evaluate based on assessment indices such as cycle availability, complexity, connectivity [1], population coverage, average edge length, station density (number of stations per square kilometer) and so on [2]. In this paper, traffic zones data of Tehran is used for designing of new ring or non-ring lines.

3. Discussion and Results

Currently, Tehran has seven subway lines and this paper propose two new ring lines (around central regions of Tehran) and four new non-ring lines (around Tehran city).

Results of FLP and FNLP models indicate in figure 1 and 2, respectively. The length of new lines designed based on FLP model is equal to 139.3 km (477000 billion Iranian Rials) while the length of new lines designed based on FNLP model is equal to 144.6 km (494000 billion Iranian Rials)

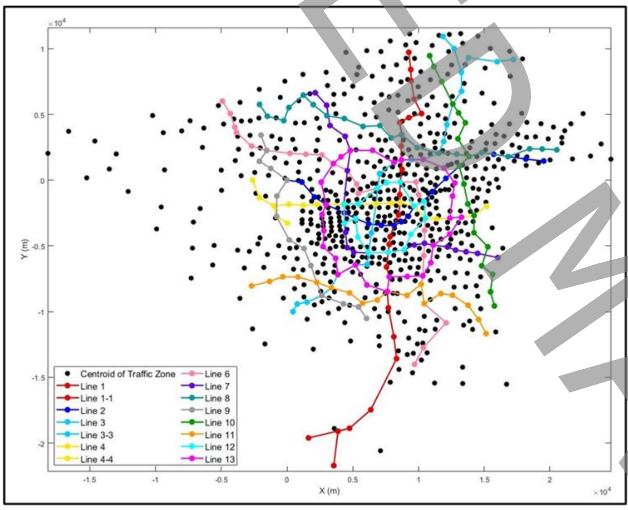

Figure 2. Designed network based on FLP model

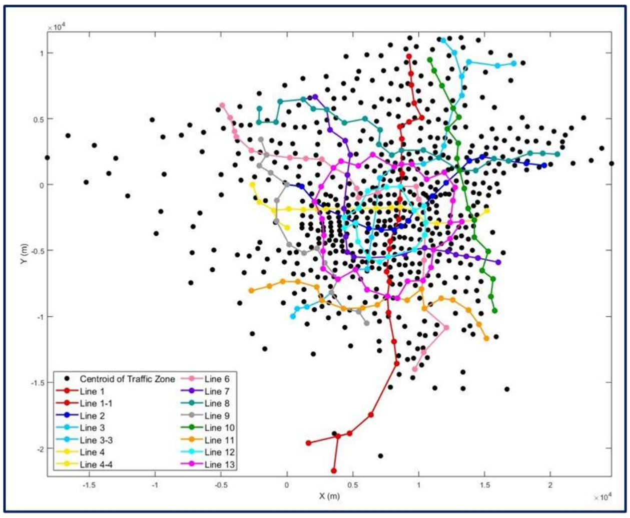

Figure 3. Designed network based on FNLP model

Values of topology improvement per construction cost (TICC) index for utilized models are represented in table 1. Results indicate that FLP model has better performance than other models.

Table 1. Values of topology improvement per construction cost index

Model	*TICC index*
LP	1.192
NLP	1.248
FLP	1.522
FNLP	1.476

4. Conclusion

In comparison to the classic deterministic approach, the proposed fuzzy approach can improve topology improvement per construction cost index by 23 percent.

5. References

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