

# Usage of Particle Filter for Exact Estimation of Constant Head Boundaries in Unconfined Aquifer

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## ABSTRACT

Having the exact values of boundary conditions is one of the effective way to develop precise groundwater models. In the present study, the exact value of constant head boundaries in Birjand aquifer is specified with the usage of particle filter linked to meshless groundwater model. Particle filter known as one of the common data assimilation methods applies to dynamic systems in order to improve performance. Meshless model, one of the numerical models that do not mesh the problem domain, enforces the governed equation to the nodes. Birjand aquifer with an almost 269 km<sup>2</sup> area, has 190 extraction and 10 observation wells. There are also nine inflow and one outflow regions with constant head boundary conditions, which include 105 boundary nodes. In this research after determination of the lower and upper bounds of groundwater head for each node, the exact values of this parameter is computed. Finally, the simulated groundwater head compared with observation data. The closeness of the achieved results to the observation data showed the performance of the engaged method, as the results indicated a significant decrease in RMSE occurs just with the usage of particle filter linked to meshless model. RMSE value reduced to 0.386 m as its previous value was around 0.757 m. Results also showed that the model was more accurate when the number of particles in particle filter were increased. The RMSE value for 500, 700 and 1000 particles were 0.484, 0.401 and 0.386m respectively.

## KEYWORDS

Birjand aquifer, RMSE, Constant Head Boundary Condition, Particle Filter, Meshless Groundwater Flow Model.

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## 1. Introduction

Groundwater is one of the main sources of fresh water in all regions and the only source of water supply in arid and semi-arid regions of the world. Nowadays, due to the excessive extraction of groundwater, these resources endanger intensely [1]. Therefore, Groundwater flow simulation in aquifers is the best way to recognize its behavior undoubtedly. Many numerical methods have been used for this aim. Meshless local Petrov-Galerkin (MLPG) categorized as a weak form method is used in this study. The independency of this method removes the drawbacks of mesh based methods e.g. finite difference method (FDM) and finite element method (FEM) [2]. Besides the groundwater simulation procedure, uncertainty assessment must be considered as well. Many researches is carried out with the purpose of uncertainty assessment. Hamraz et al. (2015), Abedini et al. (2017) and Du et al. (2018) used GMS software and GLUE method in order to simulate groundwater flow and assessment the uncertain parameters.

In the present study, a new method i.e. particle filter known as the online calibration method is linked to the meshless local Petrov-Galerkin simulation model to find the optimal values of constant head boundaries. The purpose of this study is to improve the accuracy of simulation results. This model is used for the first time in this field.

## 2. Methodology

### 2.1 Particle Filter

A particle filter known as a powerful estimation method computes the probability density function of a random process and also estimates the exact state of the object in the future time based on the states and observations of previous times. The particle filter makes some estimates for the state of the object to select the best one [3].

To this end, in the initial step, particles are scattered in the space which its dimension equals to the number of parameters that must be estimated. Each particle is assigned with a weight value. This weight value in the first step is equal for all particles around  $(\frac{1}{N})$  ( $N$  is the number of the particles scattered in state space) [4]. In the next step, the weight value depending on the position of the particle is updated. Once the weight of each particle is determined, to prevent from degeneracy occurrence, re-sampling method is carried out.

### 2.2 Meshless Local Petrov-Galerkin (MLPG)

MLPG is a weak form of meshless methods presented in 1998 by Atluri and Zhu (1998) to solve the potential

equation. This method is generally used in fluid mechanics and involves two functions: a weight function (cubic spline) and a moving kriging.

### 2.3 Groundwater flow equation in unconfined aquifer

Based on the Dupouit assumption, the governed equation of groundwater flow in a transient condition is stated in Eq. 1 [5]:

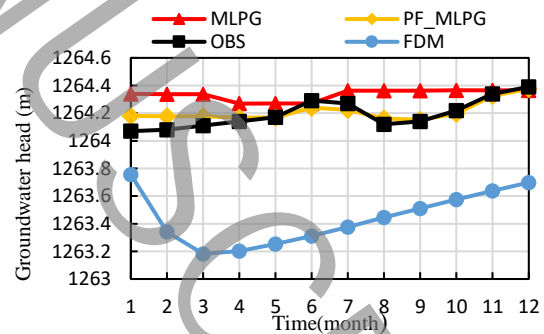
$$\frac{\partial}{\partial x} \left( k_x H \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y H \frac{\partial H}{\partial y} \right) = \frac{S_y \partial H}{\partial t} + Q \times \delta(x - x_w)(y - y_w) + q \quad (1)$$

Here,  $H$  represents groundwater head [L],  $k$  stands for hydraulic conductivity coefficient [L/T],  $Q$  denotes the discharge (+) or recharge (-) rate [L/T], and  $q$  stands for the distributed flow, e.g. precipitation and evaporation.  $S_y$  is specific yield.

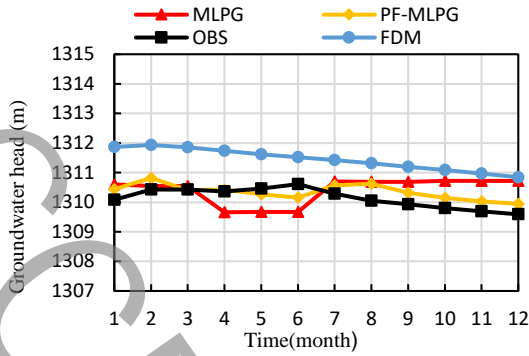
## 3. Discussion and Results

Solving groundwater partial differential equation requires precise values of constant head boundaries. Therefore, determination of the exact values of these boundaries is one of the fundamentals steps in groundwater studies. Particle filter method linked to the MLPG flow model that is calibrated and verified by the authors in the previous studies [6].

Finally, the optimal values of constant head boundaries are obtained, and, the groundwater table is computed using the achieved optimal values. Figure 1. shows the results of PF-MLPG, MLPG, FDM and observation data. In figure 1, the graphs show the high correspondence of PF-MLPG method (yellow line) to the observation data (black line). This fact clearly indicate the high accuracy of PF-MLPG method due to the usage of optimal values for constant head boundaries.



(a)



(b)

**Figure 1. Comparison of results in different methods**

In order to evaluate the performance of PF-MLPG method, mean, absolute mean, root mean square error calculated by equation (2-4):

$$ME = \frac{\sum_{j=1}^m \sum_{i=1}^n (h_o - h_s)}{m \times n} \quad (2)$$

$$MAE = \frac{\sum_{j=1}^m \sum_{i=1}^n |h_o - h_s|}{m \times n} \quad (3)$$

$$RMSE = \sqrt{\frac{\sum_{j=1}^m \sum_{i=1}^n (h_o - h_s)^2}{m \times n}} \quad (4)$$

where  $h_{oi}^t$ ,  $h_{si}^t$ ,  $\bar{h}_o$  are the level of observed groundwater and the simulated and mean of the observed values, respectively,  $n$  and  $m$  are the number of piezometers and the number of time steps. Table 1 shows the achieved results.

**Table 1. Calculation of errors**

	PF-MLPG (1000 particles) (m)	FDM(m)	MLPG (m)
Mean error (m)	-0.061	0.159	-0.12
Mean Absolute error (m)	0.298	1.434	0.573
Root mean square error (m)	0.386	1.197	0.757

RMSE is the main index for evaluation of accuracy [9]. Based on the table 1 results of PF-MLPG is more accurate than FDM and MLPG methods due to its lower value of RMSE. Also the performance of PF-MLPG method with different number of particles is investigated in Table 2. RMSE value decreases while the number of particles increases. The model also is ran for 2000 particles. However, its results are as the same of 1000 particles.

**Table 2- Calculation of errors**

	PF-MLPG (500 particles) (m)	PF-MLPG (700 particles) (m)
Mean error (m)	-0.101	-0.083
Mean Absolute error (m)	0.416	0.332
Root mean square error (m)	0.484	0.401

## 4. Conclusions

Particle filter known as one of the data assimilation method, is linked to the meshless local Petrov-Galerkin flow model to find the best values of constant head boundaries of a real aquifer. In the first step, a set of particles with the same weight values ( $\frac{1}{N}$ ) are generated in the state space. The dimension of state space are equal to the number of uncertain parameters. The studied region is Birjand unconfined aquifer which is located in South-Khorasan province. Finally, the optimal values of constant head boundaries for boundary nodes are obtained. Mean, mean absolute and root mean square error indices is calculated for PF-MLPG, MLPG and FDM method. The RMS error are 0.386, 0.757 and 1.197m for PF-MLPG, MLPG and FDM respectively. The results also reveal that with increasing the number of particles, the RMS error decreases.

## 5. References

- [1] Nayak, P., Satyaji Rao Y., Sudheer, K., 2006. "Groundwater level forecasting in a shallow aquifer using artificial," *Water Resources Management*, 20, pp. 77-90.
- [2] Liu, G. R., Gu Y. T., 2005, *An introduction to Meshfree Methods and Their Programming*, Singapore: Springer.
- [3] Hamraz, B. S., Akbarpour, A., Pourreza Bilondi M., Sadeghi Tabas, S., 2015. "On the assessment of ground water parameter uncertainty over an arid aquifer," *Arabian Journal of Geosciences*, 8, pp. 10759-10773.
- [4] Abedini, M., Ziai, A. N., Shafiei, M., Ghahraman, B., Ansari H, Meshkini, J., 2017. "Uncertainty Assessment of Groundwater Flow Modeling by Using Generalized Likelihood Uncertainty Estimation Method (Case Study: Bojnourd)," *Iranian Journal of Irrigation and Drainage*, 10(6), pp. 755-769.
- [5] Du, X., Lu, X., Hou J., Ye X., 2018. "Improving the Reliability of Numerical Groundwater Modeling in a Data-Sparse Region," *Water*, 10(3), pp. 289-304.
- [6] Arulampalam, S., Maskell, S., Gordon N., Clapp, T., 2002. "tutorial on particle filters for Online nonlinear/nongaussian Bayesian tracking," *IEEE Transaction Signal Process*, 50(2), pp. 174-188.
- [7] Fearnhead P., Kuensch, H. R., 2018. "Particle Filters and Data Assimilation," *Annual Review of Statistics and Its Application*, 5(1), pp. 421-449.
- [8] Dupuit, J., 1863. *Estudes Theoriques et Pratiques sur le Mouvement desEaux*, Paris: Dunod.
- [9] Mohtashami, A., Akbarpour A., Mollazadeh, M., 2017. "Development of two dimensional groundwater flow simulation model using meshless method based on MLS approximation function in unconfined aquifer in transient state," *Journal of Hydroinformatics*, 19(5), pp. 640-652.