

Nonlinear free vibration analysis of granular soil layer using perturbation technique

Ali Shirzad¹, Amir Hamidi², Seyed Ali Asghar Hosseini^{3*}

¹ Master of Geotechnical Engineering, Faculty of Engineering, Kharazmi University, Tehran, Iran

² Professor, Faculty of Engineering, Kharazmi University, Tehran, Iran

³ Associate Professor, Faculty of Engineering, Kharazmi University, Tehran, Iran

ABSTRACT

In this study, an experimental model has been proposed to determine the dynamic deformation properties of cemented and non-cemented granular soils and then the natural frequency of one-layered, homogeneous and horizontal surface alluvium under the influence of one-dimensional harmonic vibrations was studied. The proposed model is very compatible with laboratory results in a wide range of grain soils. The natural frequency of a one-degree-of-freedom system was determined analytically, and the results show that it has the careful accuracy. Analytical method In order to determine the response of a one-degree-of-freedom system, it has a very good agreement with the numerical method such as Runge-Kutta method. In the present study, considering the one-layered alluvium as a lumped mass system and nonlinear spring and nonlinear damping, a clear solution of this system of a one-degree-of-freedom has been proposed. On the other hand, the natural frequency can not only be a function of the depth of the alluvium layer and can be considered as a function of time.

KEYWORDS

Perturbation techniques, natural frequency, nonlinear effects, experimental formulations, one-layer deposit surface response

* Seyed Ali Asghar Hosseini : ali.hosseini@khu.ac.ir

1. Introduction

The response of the soil layer during seismic vibrations is affected by local soil conditions. Recent Destructive earthquakes have shown that topography, bedrock nature, and the nature and geometry of sedimentary soils are the main factors that have a significant effects on soil layer excitation. The characteristics of the local soil can affect the characteristics of the earthquake force on the structure.

Deep deposits of primarily dense granular material can significantly amplify ground motions. The 1967 Caracas, Venezuela earthquake, which provided undisputed evidence of the effect of "local soil conditions" on structural response, was also the first event to focus attention on the amplification potential of stiff soils [1].

The perturbation method has been used to analyse the ground's response as a direct solution of the equation of motion of one-degree-of-freedom in the time domain to determine the natural frequency of a one-layer system. One of the strongest methods in this technique is the Multiple Scales method, which has been widely used in nonlinear vibration problems [2].

The references ([3]-[6]) were able to convert nonlinear equations into a set of linear equations using the perturbation parameters. This method has been widely used to calculate the problem in geotechnical engineering.

The behavior of soil layers against seismic excitations on bedrock can be linear or nonlinear. Assessing the ground's response to nonlinear methods requires appropriate information about soil properties, and on the other hand, it requires appropriate methods that can analyse and evaluate the mathematical model of the soil layer.

In this paper, by creating a suitable mathematical model to determine the values of G/G_{max} and D , it is tried to form a differential equation governing the one-dimensional motion of the site in free vibration mode by considering the nonlinear effects of soil layer on the properties of dynamic deformation. Then, the equation governing the one-degree-of-freedom system is analysed using the perturbation technique (multiple time scale method), and as a result, the non-linear natural frequency of the one-degree-of-freedom system is determined in analytical form. The response obtained from the perturbation technique was compared with the response obtained from the Runge-Kutta method.

2. Methodology

The equation governing the mean curve behavior of the damping ratio and the normalized shear modulus for Simante and non-Simante soil can be introduced as follows:

$$G / G_{max} = (a * \gamma + b) / (\gamma + c) \quad (1)$$

$$D (\%) = (a * \gamma + b) / (\gamma + c) \quad (2)$$

The perturbation method is inherently based on the presence of small and large parameters in the problem, known as the perturbation values. In other words, the

perturbation method uses the perturbation values to convert nonlinear problems to a certain number of linear problems so that it can solve the nonlinear problem as a set of solved linear problems. In fact, the perturb parameters are the basis of this method. In addition, the multiple scale method uses multiple time scales to convert nonlinear to linear equations.

The equation for the dynamic equilibrium governing the motion of one-degree-of-freedom system, in its dimensionless form, will be as follows:

$$\ddot{u} + [\mu_2 u^2 + \epsilon \mu_1 u + \epsilon^2 \mu_0] \dot{u} + [S_3 u^3 + S_2 u^2 + S_1 u] = 0 \quad (3)$$

In order to obtain an approximate complete response of (u) with a series of small parameters (ϵ) we will have the following:

$$u(T_0, T_1, T_2) = \epsilon u_1(T_0, T_1, T_2) + \epsilon^2 u_2(T_0, T_1, T_2) + \epsilon^3 u_3(T_0, T_1, T_2) \quad (4)$$

3. Discussion and Results

The response of the nonlinear amplitude and the phase angle are considered in the equation form (5) by considering the nonlinear effects:

$$a(T_1) = \frac{2 \sqrt{\frac{(a_0^2 \mu_2 + 4 \mu_0) e^{\mu_0 T_1} - \mu_2}{a_0^2} \mu_0}}{(a_0^2 \mu_2 + 4 \mu_0) e^{\mu_0 T_1} - \mu_2} \mu_0 \quad (5)$$

$$\beta(T_2) = \frac{1}{6 \mu_2} \left[(-10 S_2^2 + 9 S_3) \ln \left[\frac{(a_0^2 \mu_2 + 4 \mu_0) e^{\mu_0 T_2} - \mu_2}{a_0^2} \right] + (10 S_2^2 - 9 S_3) \ln \left(\frac{\mu_0}{a_0^2} \right) \right. \\ \left. + (10 S_2^2 - 9 S_3) \ln(e^{\mu_0 T_2}) + (20 S_2^2 - 18 S_3) \ln(2) + 6 \beta_0 \mu_2 \right]$$

The analytical response is approximately the response of the alluvium substrate with the assumption of nonlinear stiffness and damping. To get the natural nonlinear frequency and free vibration response of the site, it is enough to have accurate information about the condition of the problem. In Figure 2, numerical solutions by integrating the equation of motion using the Runge-Kutta method were compared with the analytical solution by using the perturbation method. Figure 2 shows the results of the site's one-dimensional nonlinear response analysis for one-layer alluvium by perturbation method and the solution of the differential equation of motion by using the Runge-Kutta method in free vibration mode so that they are in good agreement with each other. The natural frequency can be considered variable according to its concept and the nature of the soil layer that changes in stiffness and damping during vibration.

$$\omega = \frac{1}{6t \mu_2} \left[\begin{array}{l} 10 \ln(4 e^{\mu_0 t}) S_2^2 - 10 \ln \left(\frac{e^{\mu_0 t} (a_0^2 \mu_2 + 4 \mu_0) - a_0^2 \mu_2}{a_0^2} \right) S_2^2 \\ + 10 \ln \left(\frac{\mu_0}{a_0^2} \right) S_2^2 - 9 \ln(4 e^{\mu_0 t}) S_3 \\ + 9 \ln \left(\frac{e^{\mu_0 t} (a_0^2 \mu_2 + 4 \mu_0) - a_0^2 \mu_2}{a_0^2} \right) S_3 - 9 \ln \left(\frac{\mu_0}{a_0^2} \right) S_3 \\ + 6t \mu_2 \end{array} \right] \quad (6)$$

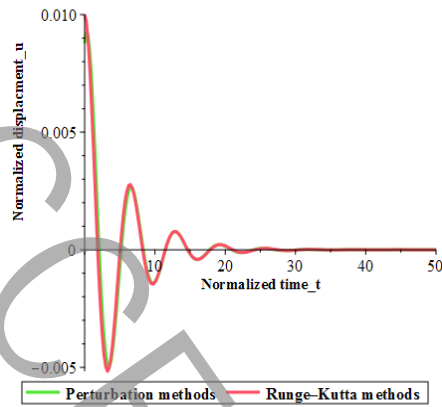


Figure 2. Comparison of the results of non-linear site response analysis with numerical integration methods of equation of motion and analytical method of perturbation

4. Conclusions

In this paper, perturbation method is used to provide analytical formulation to determine the surface response of a one-layer grain alluvium by considering hysteresis and nonlinear effects during free vibration. In addition, the natural frequency of the alluvium layer, in addition to the effect of parameters such as confining pressure (P_0), is a function of the time of vibrations, and the natural frequency becomes convergent to one after a sufficient time. Also, in order to determine the natural frequency of the alluvium layer based on the specifications of the desired layer, an analytical formulation was provided for it. On the other hand, an experimental formulation based on nonlinear regression of experimental data has been used to estimate the damping ratio and shear stiffness of cemented and non-cemented granular soils. The formulation used using the Taylor series to the approximately the third time around the zero point has become a polynomial with respect to the u-displacement. For many studies, this polynomial approximates the shear stiffness and damping ratio for

most previous studies of researchers to the shear strain of 0.1%. In order to verify, the perturbation method was compared with the Runge-Kutta method and it was observed that the perturbation method with the Runge-Kutta method in free vibration mode has a very good agreement. In order to perform one-dimensional analysis and provide analytical formulation for site response and natural frequency and response amplitude of alluvium layer, it was assumed that the desired layer is a semi-infinite, homogeneous and non-elastic space. On the other hand, it was assumed that $\partial u / \partial y$ was linear and that the deformations in the vertical direction were not considered.

The most important results are as follows:

1. Using regression analysis, the experimental model to estimate and evaluate the shear modulus and the damping ratio of cemented and non-cemented soils was established. In order to consider the effect of cement percentage, the void ratio, the confining pressure were determined using three constants.
2. Using the perturbation technique, it was found that it would be possible to determine the response of the one-layer alluvium surface as a one-degree-of-freedom system with a nonlinear spring and a nonlinear damper to a satisfactory extent. By comparing the results of the analysis with the perturbation method and the numerical method, it is possible to understand this issue.
3. As the depth of the alluvial layer increases, the natural frequency decreases. This change in natural frequency is in very good compatibility with the relationship obtained by solving the wave propagation equation in the homogeneous elastic medium.

5. References

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