Presentation a New Method in Mathematical Modeling of Pollutant Transport in Rivers with Storage Zones

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ABSTRACT

Prediction of pollutants transport in water resources is of particular importance in the management and prevention of their pollution. The heterogeneity and non-uniformity in the morphology throughout rivers which known as the storage area, will make changes in the uniform transport of pollutants to the downstream. Storage areas along rivers are actually places around river where flow velocity in this places is significantly slower than river's flow velocity and are also known as dead zones. The presence of these places in rivers makes it difficult to apply the classic pollutant transport equation for them. For more accurate simulation of the pollutant transport in natural rivers containing storage zones, some improvements should be made to the classic advection-dispersion equation. In this study, a new approach is presented by considering nonlinear flux dispersion and applying storage zones. In order for verification and validation of the proposed model, two series of hypothetical and real data examples have been used. Based on the measured results, the model outputs have acceptable adaptation with observational data and shows that the proposed model is an accurate and acceptable model in simulation of dissolved pollutant transport in rivers with storage zones. According to the obtained concentration-time curves, it can be concluded that the proposed model can model any type of storage area with any amount of area, also this model is applicable for all rivers with and without storage area and it is more superior in comparison with other similar models in terms of number of parameters (considering merely one parameter) and simplicity in physical interpretation; and can be an appropriate alternative instead of the classic pollutant transport model in these type of rivers.

KEYWORDS

Advection-Dispersion Equation, Storage zone, Non-linear Flux, Dead Zone, Dispersion Flux.

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1. Introduction

Researchers have been conducting various experiments with tracer materials and simulating the solute transport in rivers with storage zones using the classic Advection-Dispersion Equation (which not consider mass exchange between these areas and main channel). They observed that the results of the equation do not correspond well with the experimental data; and the real data and simulation results have different characteristics. Including that the real data at downstream show a lower peak and a longer sequence in the concentration-time diagram.

After simulating process and considering storage zone, the researchers observed a good agreement between the simulation data and real data. And they proposed several approaches for more accurate simulation and error reduction.

One of the common methods for simulating the dissolved matter transport in rivers with storage zones is the Transient Storage Model (TSM), which is able to study the exchange of pollutants between the main waterway and storage zones. This model calculates the pollutant concentration in the main waterway and storage zone, and the storage zone is indicated by the mass exchange due to the difference between the concentrations of the rivers and the storage zone. In addition, significant portions of the water may enter pebbles and coarse sands of the river bed or porous areas along the river banks. [1]

Also, the Fractional Advection-Dispersion Equation model was presented to simulate the transport of dissolved matter in rivers with storage zones. The tracer material is dispersed by different jumping behaviors. The skewness in the concentration-time curve of the dissolved material is due to the slow-moving component of transport phenomenon (e.g., dead zones or substrate saturation zones), which is not determined by the normal advection-dispersion equation. Due to the irregular behavior of pollutant particles in aquatic environments, researchers have concluded that by using fractional derivatives, it is possible to more accurately estimate the distribution of pollution in aquatic environments; and this shortcoming in the fractional advection-dispersion equation was removed by adding $\frac{\partial}{\partial x}(AD\frac{\partial^{\alpha}C}{\partial x^{\alpha}})$ term. In 2004, fractional advection-dispersion model was proposed for modelling the storage zone effects. [2]

Another common method for simulating dissolved matter transport in rivers with storage zones is the Variable Residence Time model, which simulates the advection and dispersion of dissolved matter in natural flows. The variable residence time model is a powerful and flexible numerical tool for simulating solute transport in natural flows.

In the present study, we try to increase the accuracy of the transport equation in simulation of pollutant transport in rivers with storage zones by using a simple and effective approach with minimal changes in the classic advection-dispersion equation. In this research, the main purpose is to preserve the practical aspects of the method and for applying the effect of storage zones, only one additional parameter has been used in the classic advection-dispersion equation in order to minimize the number of input parameters in the model in terms of storage zone effect. Also attempts are being made to explain the applied physically changes and in accordance with the physics of the transport problem.

Table 1. Comparison between famous models in the field of pollutant transport simulation

Model title	Number of equations	Number of dependent variables in equations	Number of parameters in terms of applying storage zone	Considering the physics of the storage zone	Considering the area properties of the storage zone
Classic ADE	1	1	0	×	×
TSM	2	2	2	✓	✓
FADE	1	1	1	×	×

2. Materials and Methods

The equation governing the pollutant transport phenomenon in rivers is the Advection-Dispersion Equation, which is a type of parabolic partial differential equations and is obtained by combination of continuity equation and Fick's first law. The classic Advection-Dispersion equation is expressed as Equation 1:

$$\frac{\partial\left(AC\right)}{\partial t}=-\frac{\partial\left(QC\right)}{\partial x}+\frac{\partial}{\partial x}\left(AD\frac{\partial C}{\partial x}\right)-AKC+AS \qquad (1)$$

In the above Equation, A is flow area, C is pollutant concentration, Q is flow discharge, D is dispersion coefficient, K is decay coefficient, S is source term, t is time variable and x is location variable. [3]

2.1. Mass transport processes

The processes of mass transport in river are the same processes of advection and dispersion. The process of mass advection occurs by the motion of the flow and the process of dispersion occurs due to the velocity gradient in river. The transported mass flux by the advection and dispersion processes are obtained using Equations 2 and 3, respectively:

$$J_{\mathrm{Adv}}=UC \qquad (2)$$

$$J_{\mathrm{Dip}}=-D\frac{\partial C}{\partial x} \qquad (3)$$

Which J_{Adv} advection flux and J_{Dip} is dispersion flux. [4]

As a matter of fact, the storage parameter indicates the presence of the storage zone in a part of river length and its value also depends on the extent and geometry of the storage zone. In this case, the general form of the advection-dispersion equation for unsteady and non-uniform flow condition will be as follows:

$$\frac{\partial\left(AC\right)}{\partial t}=-\frac{\partial\left(QC\right)}{\partial x}+\frac{\partial}{\partial x}\left(AD\left(\frac{\partial C}{\partial x}\right)^{\beta}\right)-AKC+AS \qquad (4)$$

For the steady and uniform flow condition, Equation 4 is simplified as follows:

$$\frac{\partial c}{\partial t}=-V\times\frac{\partial c}{\partial x}+\frac{1}{A}\times\frac{\partial}{\partial x}\left(AD\left(\frac{\partial C}{\partial x}\right)^{\beta}\right)-KC+S \qquad (5)$$

According to the presented equations, in the case of $\beta = 1$, Equation 5 becomes a classic advection-dispersion equation and is used for parts of river without storage zone.

The main strength point of the presented method in this research for modeling storage zones compared to other methods is the use of a minimum number of parameters (only one parameter) as well as simplicity in its physical interpretation. It should be noted that in other approaches of simulation of storage zones, the simulation process is mainly more complex than the method which presented in this research. In this research, the Method of Line has been used to solve Equation 5, which is a method for solving time-dependent partial differential equations.

3. Results and Discussion

The application of the proposed model in the field of pollutant transport simulation will be shown in the form of a hypothetical example as well as real test case. In order to demonstrate the advantages and features of the proposed numerical model, also the results of its runs will be compared with the results of the classic advection-dispersion equation, both in the case of the hypothetical and real examples.

Table 2. Test cases information

Test case	River length	Flow condition	Upstream BC for pollutant transport	location
Test case 1	10 km	steady and uniform	Continuous for 1.5 hours	Hypothetical
Test case 2	433 m	steady and non-uniform	Continuous for 3 hours	US - California

To run the transport model, one initial condition and two boundary conditions are required. The initial concentration of pollutant in the main channel will be considered as the initial condition. The upstream boundary condition is the pattern of pollutant loading which enters the river, also the downstream boundary condition will be considered as zero gradient boundary type.

Dispersion coefficient is known as one of the most important parameters of the pollutant transport modeling. In the real test case, this coefficient is obtained via field measurements (dispersion coefficient calculated by hydraulic and hydrodynamic characteristics measurements) and in the presented hypothetical example similar to the real test case, the value of the dispersion coefficient was calculated by assumed hydraulic and hydrodynamic characteristics.

3.1. Test case 2

In fact, the main purpose of designing this example is to show the concept of transient storage in the form of a comparison of concentration-time diagrams. Another aim is to show the effect of the β parameter to consider the mass exchanges between the storage zone and the main channel and also the relationship of the storage zone with different β values.

In the hypothetical example, to show the effect of storage zones, the results are presented for different values of storage parameter. Higher values of this parameter are used to create longer sequences and more skewed curves which indicating that the dissolved material will move to the downstream much more slowly than expected before. Thus higher values of storage parameter is required.

4. Conclusion

In this study, a comprehensive model for numerical solution of advection-dispersion equations in rivers with and without storage zones was developed to eliminate the shortcomings of current models in the field of pollutant transport simulation and also it be simpler in terms of application. Also, the number of input parameters in the model for prediction the effect of the storage zone is minimized to make it easier to use. Moreover, the model run time and the calibration time of the input parameters for storage zone is minimized.

These cases show the remarkable ability of the proposed model to predict pollutant transport in natural rivers and the proposed model can be used to predict the spatial and temporal pollutant concentration distribution.

References

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