

# Jump of Circles: A new way to solve the engineering optimization problems

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**Abstract:** In this paper, a new meta-heuristic optimization method called the Jump of Circles Optimization Method is introduced. In any optimization problem, an answer zone is defined in which the optimization algorithms search the space to find the optimal answer. The method presented in this paper, uses two important pillars in searching the answer zone. The first pillar is to use the geometric principles. The Jump of Circles, uses the circle with decreasing radius. The second pillar is to use the meta-heuristic application. In meta-heuristic algorithms, the search points distribute randomly and jump in answer zone. In the proposed method, the center of the searching circle jumps and sits on the optimal point of each step. The proposed algorithm solves the optimization problem in two phases. The first phase is optimal area exploration and the second phase is exploiting the exploration. Finally, the most optimal point that will be obtained from the two phases, is the optimal answer of the problem.

**Keywords:** Jump of Circles Optimization Method, meta-heuristic method, two phases method, optimization, engineering benchmark problems.

## 1- Introduction

Optimization is a very applicable tool used in almost all sciences. With this highly powerful tool, any problem that can be shown mathematically can be solved. High applicability and problem solving capability has increased the importance of optimization among researchers in recent years. The Genetic Algorithm (GA) is a highly applicable algorithm used in many optimization problems [1]. Such other researches are Particle Swarm Optimization method (PSO)[2], Ant Colony Optimization method (ACO)[3] and Gravitational Search Algorithm (GSA)[4].

There are two different phases in the “Jump of Circles Optimization Method (JCOM)” presented in this paper: in phase 1, effort is made to identify a part in the answer zone where there is the most optimal answer and then in phase 2, use is made of that answer to obtain the problem’s optimal answer. The two mentioned phases somehow overlap one another to find the better answers, that is, at the beginning of phase 2, the radius of the circle is so chosen that it may exceed that of the final limit in phase 1. To investigate different parts of the answer zone, use has been made, in the proposed method, of random numbers to select the study points. It is worth noting that the random points, generated by the algorithm in both phases, face circles that get smaller and smaller.

## 2- Methodology

This section explains the “Jump of Circles Optimization Method”. It is worth mentioning that the center of the circle is displaced in each step called the jump of circles. Since many of problems are constrained, use has been made of an evaluator function as follows to evaluate points [5]:

$$EF = GF \left( 1 \pm r_p \cdot CV^2 \right) \quad (1)$$

$$r_p = 4 \left( 1.6^{\left[ \frac{i}{2} \right]} \right)$$

Where,  $EF$  is the Evaluator Function,  $GF$  is the Goal Function,  $CV$  is the Constraint Violation and  $r_p$  is the penalty coefficient of the constraint violation in step  $i$  (it is a very useful factor used to find accurate answers). This function is based on statistical studies,  $[\ ]$  shows the integer part, and the plus and minus signs are used for the minimization and maximization, respectively. The proposed method performs the optimization in two phases as follows.

### 2-1- Phase 1: Exploring the optimal area

In the first step, a point is selected in the answer zone quite randomly and then some random search points are created around it with a limited distance  $r$  which is the radius of the hypothetical circle reduced in each step according to the following linear relationship:

$$r_0 = 0.1 \left( x_{max}^d - x_{min}^d \right) \quad (2)$$

$$r_i = r_0 - r_0 \times \frac{i-1}{i_{max1}} \quad (3)$$

$$x_{i_{new}}^d = x_{i-1}^d + ab.r_i \quad (4)$$

Where,  $x_{max}$  and  $x_{min}$  are the maximum and minimum values extractable for the desired variable, respectively,  $r_0$  is the initial radius of the circle,  $r_i$  is the radius of the circle in step  $i$ ,  $i_{max1}$  is the maximum number of steps in phase 1,  $x_i^d$  is the position of the random point in step  $i$  and in direction  $d$ ,  $x_{i_{new}}^d$  is the position of the new search point in step  $i$  and in direction  $d$ ,  $a$  is a random sign and  $b$  is a random number between 0 and 1. Next, situations of the search points are evaluated by the evaluator function and the three best ones are selected; in minimization problems (truss weight), the best situation is when  $EF$  has the least answer. The “jump of circles” algorithm considers two hypothetical lines between the best and two other points and then generates two hypothetical points on each line so that they may lie on both sides of the best point:

$$\Delta x = x_0 \cdot \left( \frac{i_{max1}}{i-1} \right) \cdot \left( \frac{x_{max} - x_{min}}{2 \times i_{max1}} \right) \quad (5)$$

$$x_{i_{new}}^d = x_{i_{best}}^d \pm \Delta x \quad (6)$$

Here,  $x_0$  is a random number between 0 and  $\pi$ , the expression in parentheses in the right side of relation (5) is to fit the value of  $\Delta x$  to the problem dimensions (it is worth mentioning that the earlier searches need more random values for the algorithm’s better answers, but the more the solution continues, the less will be this need; this has been considered in the solution through the expression in parentheses in the left side of relation (5)), and  $x_{i_{best}}^d$  is the situation of the best point in step  $i$  and in direction  $d$ . The situations of the four new points are estimated by the evaluator function; if there is any improvement in a best point, it will be considered as the step’s best point. To start the next step, the center of the circle is positioned on the best point of the last step instead of on the random point of the first step; this will continue until phase 1 stops.

### 2-2- Phase 2: Exploiting the exploration

The best point of the last step in phase 1 is considered as the center of the circle at the beginning of phase 2. In order for the circle concept to really enter the solution, building relations in this phase of the problem is done circularly in two consecutive dimensions; in other words, the relationships in the even dimension depend on those of the former odd dimension. In phase 2, some search points are created inside the hypothetical circle and some on its periphery as follows:

$$x_{i_{new}}^{d=2m-1} = x_{i_{center}}^{d=2m-1} + r_i \times \cos(\alpha) \quad (7)$$

$$x_{i_{new}}^{d=2m} = x_{i_{center}}^{d=2m} + r_i \times \sin(\alpha) \quad (8)$$

Where,  $2m-1$  shows the odd direction and  $2m$  represents the even one,  $\alpha$  is the angle of the desired point to be chosen from the  $[0-2\pi]$  interval divided equally among the points on the periphery (we suggest 4-6 points on the periphery where speed is important and 24-30 where accuracy is vital), and  $r$  is the radius of the circle reduced linearly in each step as follows:

$$r_0 = 0.03(x_{max}^d - x_{min}^d) \quad (9)$$

$$r_i = r_0 - r_0 \times \frac{i-1}{i_{max2}} \quad (10)$$

Where,  $x_{max}$  and  $x_{min}$  are, respectively, the maximum and minimum values extractable for the desired variable,  $r_0$  is the initial radius of the circle in phase 2,  $r_i$  is the radius of the circle in step  $i$  of phase 2, and  $i_{max2}$  is the maximum number of steps in phase 2. It is worth noting that  $r_i$  reaches zero at the end of phase 1, but it begins phase 2 with about 1/3 of the initial value in phase 1; this causes the overlapping of the two phases and helps to improve the answers. In the “jump of circles”

algorithm, the number of the search points of the circle is a multiple of 4 created by the following relations in such a way that their number is equal in each quadrant.

$$x_{i_{new\ 1}}^{d=2m-1} = x_{i_{center}}^{d=2m-1} + a4 \times (a1 \times r), x_{i_{new\ 1}}^{d=2m} = x_{i_{center}}^{d=2m} + a4 \times \sqrt{r^2 - (a1 \times r)^2} \quad (11)$$

$$x_{i_{new\ 2}}^{d=2m-1} = x_{i_{center}}^{d=2m-1} + a3 \times (a2 \times r), x_{i_{new\ 2}}^{d=2m} = x_{i_{center}}^{d=2m} - a3 \times \sqrt{r^2 - (a2 \times r)^2} \quad (12)$$

$$x_{i_{new\ 3}}^{d=2m-1} = x_{i_{center}}^{d=2m-1} - a2 \times (a3 \times r), x_{i_{new\ 3}}^{d=2m} = x_{i_{center}}^{d=2m} + a2 \times \sqrt{r^2 - (a3 \times r)^2} \quad (13)$$

$$x_{i_{new\ 4}}^{d=2m-1} = x_{i_{center}}^{d=2m-1} - a1 \times (a4 \times r), x_{i_{new\ 4}}^{d=2m} = x_{i_{center}}^{d=2m} - a1 \times \sqrt{r^2 - (a4 \times r)^2} \quad (14)$$

$a1$ ,  $a2$ ,  $a3$  and  $a4$  are four random numbers in the  $[0,1]$  interval and the value of the evaluator function is calculated at the search points created on the periphery of the circle and inside it; this information is then used to create the other four search points. To create the new points, we first specify half of the set-close points on the circle periphery whose sum of the evaluator function is the best value; in maximization problems, it is the maximum value and vice versa. Next, the exit direction from the circle is considered toward this half and four points with the best situations are selected from among all the search points created inside and on the circle periphery. Since these points are in the direction of the circle exit or entry, they will yield a new search point.

$$\Delta x = x_0 \cdot \left( \frac{i_{max\ 2}}{i - 1} \right) \cdot \left( \frac{x_{max} - x_{min}}{2 \times i_{max\ 2}} \right) \quad (15)$$

$$\begin{cases} x_{i_{new}}^d = x_{i_{point}}^d + \Delta x \cdot dir & \text{If located in output direction} \\ x_{i_{new}}^d = x_{i_{center}}^d + \Delta x \cdot dir & \text{If located in input direction} \end{cases} \quad (16)$$

Where,  $i_{max\ 2}$  is the maximum number of iterations and  $i$  is the counter of steps in phase 2. Direction is shown by  $dir$  which is actually a positive or negative number recognized as explained earlier. Situations of the new search points are evaluated by the evaluator function and one point with the best value (from among these 4 and those taken from the circle) is considered as the best point of this step; the circle then jumps to this point for the next step and the process will continue until phase 2 stops; the best point of the last step will be the problem's optimal answer.

### 3- Conclusions

This paper introduced the "Jump of Circles Optimization Method (JCOM)" which is, in fact, a meta-heuristic method combined with geometric constraints. The proposed algorithm consists of two phases; phase 1 explores the optimal zone and phase 2 exploits the exploration. Each phase is responsible for one important optimization principle, they overlap to yield better answers, and meta-heuristic applications ease taking points from different parts of the search space.

### References

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