A micromechanical inelastic strain-damage constitutive model based on wing- and secondary-cracking mechanisms under dynamic loading

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ABSTRACT

For most rock materials, there exists a coupling between inelastic deformations caused by crack displacements on micro-crack faces and damage evolution due to nucleation and growth of wing- and secondary-cracks. While rock material is subjected to dynamic loading, the interaction between micro-cracks plays an important role in materials behavior. The self-consistent homogenization scheme is implemented in this paper to consider micro-cracks interaction and determine the equivalent mechanical properties of micro-cracked rock deteriorated by damage evolution. The aim of this article is to develop a self-consistent based micromechanical damage model by taking into account the wing- and secondary-cracking mechanisms accompanied by inelastic strains caused by crack displacements under dynamic compressive loading. While stress intensity factors in tensile and in-plane shear modes at flaw tips exceed the material fracture toughness in modes I and II, respectively, wing- and secondary-cracks are sprouted and damage evolution occurs. For closed cracks, an appropriate criterion for the secondary-crack initiation is proposed in this paper. The developed model algorithm is programmed in the commercial finite difference software environment for numerical simulation of rock material to investigate the relationship between the macroscopic mechanical behavior and the microstructure. The fracture toughness parameters of the rock samples are experimentally determined. The rock microstructure parameters (average initial length and density of flaws) are studied using scanning electron microscopy. In order to verify the developed model, a series of numerical simulations are carried out to numerically reproduce the Split-Hopkinson pressure bar test results. The simulation results demonstrate that the developed micromechanical model can adequately reproduce many features of the rock behavior such as softening in post-peak region, damage induced by wing- and secondary-cracks and irreversible deformations caused by crack displacements on micro-cracks. Furthermore, the softening behavior of rock material in the post-peak region is affected by considering inelasticity and the secondary-cracking mechanisms. Therefore, the rock sample simulation with the coupled inelastic-damage model can increase inelastic deformations in the post-peak region as a result of irreversible strains caused by crack displacements on micro-cracks. The simulation by considering the secondary-crack mechanism leads to an increase in the micro-cracking process, damage and fragmentation in rock material.

KEYWORDS

Secondary-crack, stress intensity factor, inelastic strain, self-consistent, Sungun copper mine.

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Introduction

Under dynamic compressive stress fields, inelastic deformations are high, in compared with quasi-static compressive field. Thus, various mechanisms have been suggested in the literature for high inelastic deformations under dynamic loading condition. In this paper, inelastic deformations caused by relative displacements along initial micro-cracks and wing- and secondary-cracking mechanisms are incorporated into a homogenized constitutive formulation based on the self-consistent scheme. Researchers such as Xie et al. [1] and Molladavoodi [2] used the Ponte-Castaneda and Willis or Mori-Tanaka homogenization schemes to study materials behavior with closed frictional micro-cracks. In another study, Paliwal and Ramesh [3] established a micromechanical damage model based on the self-consistent homogenization scheme accounting for two-dimensional slit-like micro-cracks embedded in an elliptical inclusion surrounded by a homogenized solid matrix. Ayyagari et al. [4] proposed a fully three-dimensional generalized anisotropic compliance tensor for brittle solids and evaluated considering the wing-cracking mechanism, using a mixed-approach based on kinematic and energetic arguments.

The aim of this study is to develop a micro-mechanical damage model to take into account damage due to the both wing- and secondary-cracking mechanisms under dynamic compressive loading. Moreover, in this paper, the model is extended to include the coupling of two dissipative mechanisms, i.e. damage evolution and inelasticity induced by relative crack displacements for the case of closed micro-cracks. The proposed coupling between inelasticity and damage evolution is formulated in the framework of a micromechanical model based on the self-consistent homogenization scheme, which is programmed and implemented into a commercial code. Accordingly, the proposed model is applied for the simulation of brittle rocks behavior under dynamic loading.

Description of the theoretical framework

According to Ayyagari et al. [4], it is assumed that sliding on flaws takes place along the long diameter of the pre-existing microcrack named as the $\underline{P}$ direction. The direction $\underline{Q}$ is transverse to the flaw plane in 3D, as illustrated in Fig.1. In this paper, the pre-existing microcrack geometry is presumed to be a planner elliptic with a long diameter of 2S and a normal orientation $\underline{N}$.

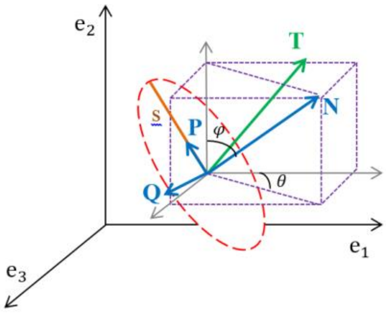

Figure 1.The 3D representation of the microcrack [8].

(P, N, Q) represent the local flaw basis defined with respect to the global coordinate system (e_1, e_2, e_3) using their direction angles. The normal orientation ($\underline{\boldsymbol{N}}$) of the pre-existing microcrack, the sliding direction ($\underline{\boldsymbol{P}}$) and the wing-crack normal orientation ($\underline{\boldsymbol{N}}_w$) can be defined in 3D space by two angles φ and θ, as illustrated in Fig.1. The crack normal ($\underline{\boldsymbol{N}}$), the sliding direction ($\underline{\boldsymbol{P}}$) and the wing-crack normal ($\underline{\boldsymbol{N}}_w$) are determined in the global coordinate system as follows [8]:

$$\underline{N} = \begin{bmatrix} \sin\varphi\cos\theta \\ \cos\varphi \\ \sin\varphi\sin\theta \end{bmatrix}, \underline{P} = \begin{bmatrix} \cos\varphi\cos\theta \\ -\sin\varphi \\ \cos\varphi\sin\theta \end{bmatrix} \qquad (1)$$

$$\underline{N}_w = \begin{bmatrix} \cos\theta \\ 0 \\ \sin\theta \end{bmatrix}$$

The inclusion stress ($\underline{\underline{\sigma}}^e$) is the stress state applied to the elliptical inclusion embedded in an elastic matrix. The inclusion stress traction on the crack plane is ($\underline{T} = \underline{\underline{\sigma}}^e.\underline{N}$). The inclusion stress traction can be decomposed into the crack normal ($\sigma_{nn} = \underline{N}.\underline{T}$) and the sliding ($\underline{T}.\underline{P}$) components, controlling cracking mechanisms. The sliding ($\underline{T}.\underline{P}$) component is the driving force for sliding along pre-existing micro-cracks and wedging (gapping) for nucleation of wing-cracks. The crack normal ($\sigma_{nn} = \underline{N}.\underline{T}$) component indicates whether the pre-existing crack grows under opening (tensile) ($\sigma_{nn} > 0$) or closed (shear) ($\sigma_{nn} < 0$) modes. The damage in rock material is due to the wing- and secondary-cracking mechanisms. Therefore, the damage parameter (Ω) can be divided into wing-crack damage (Ω^w) and secondary-crack damage (Ω^s) as follows:

$$\Omega = \Omega^{w} + \Omega^{s} \qquad (2)$$

The inelastic strain by considering the both wing- and secondary-cracking mechanisms can be written as follows:

$$\varepsilon^{in} = \eta s \left[\varpi_1 u_p \left(\underline{N} \otimes^s \underline{P} \right) + \varpi_2 u_p \left(\underline{N}_w \otimes^s \underline{P} \right) + \varpi_3 \left(\underline{N}_w \otimes^s \left(\underline{\sigma}^e . \underline{N}_w \right) \right) + \varpi_4 \Delta s \left(\underline{N} \otimes^s \underline{P} \right) \right] \quad (3)$$

Where, ϖ_1, ϖ_2, ϖ_3 and ϖ_4 are the coefficients to consider the inelastic strain in three-dimentionals scale. ΔS, is the increment in the pre-existing microcrack length due to the secondary-cracking mechanism.

Experimental studies for rock parameters determination

The studied rock is taken from the Sungun mine located in Azerbaijan Province, Northwest Iran. The Sungun mine is an open pit mine exploiting the Sungun copper porphyry deposit that is an intrusive porphyritic igneous rock. The Sungun Porphyry (SP) is the host rock of the copper mineralization forming the main lithological unit at the Sungun mine. The mechanical, micromechanical and fracture toughness parameters of the SP are listed in Table 1.

Table 1. The mechanical, micromechanical and fracture toughness parameters of the SP.

Rock properties	Value
σ_c (MPa)	37
E (GPa)	9.5
v	0.23
$K_{IC}(\mathrm{MPa}\sqrt{\mathrm{m}})$	0.77
$K_{IIC}(\mathrm{MPa}\sqrt{\mathrm{m}})$	1.4
$2s_0$ (μm)	96.5
$\eta\left(\frac{1}{\mathrm{m}^2}\right)$	5.46e8

Numerical simulation results

The main objective of the developed micromechanical damage model is to reproduce and predict the brittle rock behavior under dynamic compressive loading. The inelasticity caused by relative crack displacements, damage evolution and the sensitivity of the compressive peak strength to the applied strain rate are some key features of rock brittle behavior, which are of great interest under dynamic compressive loading. The numerical model with the width and height of 25 mm is discretized with (30*30) elements. To verify the developed micromechanical damage model, the simulated stress-strain curves for the SP are plotted against and compared with the experimental stress-strain curves of the SHPB test in Fig.2 under the same imposed strain rates.

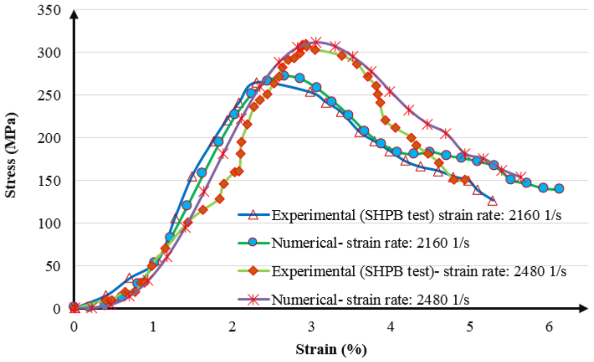

Figure 2. The comparison between the numerical and experimental results.

Conclusion

In this work, a micromechanical damage model was developed to take into account the coupling between inelasticity and the damage process under dynamic compressive loading condition. A major feature of the developed model is that the secondary-cracking mechanism, in addition to the wing-cracking mechanism is considered in the micromechanical model. Moreover, the shear mode criterion and the evolution rule for secondary-cracking based on the physical facts at the micromechanical scale were proposed in the classic fracture mechanics framework to take into account the secondary-cracking mechanism. Variation of the applied strain rate significantly affected the mechanical response of the brittle materials. Furthermore, the axial inelastic strain increment and damage evolution in rock specimen were recorded during numerical simulation by the proposed inelastic- damage micromechanical model under dynamic uniaxial compressive loading condition.

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