

# Estimation of cracking, yield, and ultimate capacity of FRP-strengthened reinforced concrete and steel sections using wavelet transform

M.R. Mohammadizadeh<sup>1\*</sup>, S. Salami<sup>2</sup>

<sup>1</sup> Assistant Professor, Department of Civil Engineering, University of Hormozgan, Bandar Abbas, Iran

<sup>2</sup> M.Sc. Graduate, Department of Civil Engineering, University of Hormozgan, Bandar Abbas, Iran

## ABSTRACT

Damage detection is a topic of great importance for structural health monitoring. Many varieties of structural damage can be detected by examining changes in structural response in terms of stiffness. Wavelet transform is a powerful mathematical tool for the processing and time-frequency analysis of transient signals and has great potential to be used in structural damage detection. In FRP-strengthened reinforced concrete and steel sections, stiffness changes can be caused by cracking, yielding of steel components, crushing of concrete, or rupture of FRP panels. With the help of wavelet transform, it is possible to use the continuous measurements of the response to bend or torsional loading to estimate the capacity of the cross section corresponding to the stiffness changes. In this paper, bending of FRP-reinforced steel beams filled by concrete under bending and CFRP-reinforced concrete beams under pure torsion is evaluated. The results show that the location of the damage appears as perturbations in the diagram of discrete wavelet coefficients, which indicate the time of cracking, yielding of steel, crushing of concrete in the compression zone, and rupture of FRP. Therefore, a wavelet transform-based data processing procedure can be used to estimate the cracking and yielding capacities of the beams subjected to torsion, the yielding capacity of the steel and the ultimate capacity of the beams subjected to bending. The results demonstrate a high level of agreement between the estimates obtained from the discrete wavelet transform method and the examined experimental and numerical data.

## KEYWORDS:

Wavelet transform, damage detection, cracking capacity, torsion, bending

---

\* Corresponding author: E-mail: [mrzmohammadizadeh@yahoo.com](mailto:mrzmohammadizadeh@yahoo.com), [mrz\\_mohammadizadeh@hormozgan.ac.ir](mailto:mrz_mohammadizadeh@hormozgan.ac.ir)

## 1. Introduction

Detection of the damages in structural retrofitting or its reconstruction is very essential [1]. It is significant that a reliable, effective and noninvasive detection method is used to maintain the safety and integrity of structures [2]. One of the damage detection methods is wavelet transform. Among the most important studies, we may refer to Wei et al. [3], Zhong and Oyadiji [4], Yang and Oyadiji [5], Patel et al. [6]. In the present study, first the wavelet transform is theoretically defined. Thereafter, it will be shown that the capacity of the reinforced beams under torsion or bending will be obtained through wavelet transform analysis.

## 2. Wavelet transform theory

Signal processing is mainly performed aiming at obtaining as much information as possible from the initial signal. Continuous wavelet transform was developed as a method based on window Fourier transform to solve its response problem. In wavelet transform, the width of the window varies to calculate each frequency range of the signal, and this is the most important characteristic of wavelet transform [7, 8]. One of the main advantages of the wavelets is the ability of local analysis of a large signal. Wavelet transform reveals aspects of the data that could not be shown by other signal analysis techniques. Aspects such as trends, breakpoints, and discontinuities can be identified through higher derivatives in wavelet analysis [9].

## 3. Behavior of structural elements under the influence of load at various phases

The behavioral diagram shown in "Figure 1" indicates the various behavioral phases of a reinforced concrete under bending or torsional loading. The OA segment of the curve shows the completely elastic behavior of the concrete section. As the load increases, the AB segment of the curve shows an elastoplastic behavior in the section. In the BC segment of the curve, the steel strain increases and the concrete collapse. Finally, point C represents the failure of the section and the corresponding force indicates the ultimate capacity of the concrete section. The phase shift always accompanies lower hardness in the section under the influence of loading. Generally, the capacity-demand curves of the reinforced concrete elements under the influence of bending or torsion is as shown in "Figure 1" By examining the capacity-demand curve and discrete wavelet transform, we can track the cracking, yielding and ultimate capacities of the section with the increased loading in the loading regime.

## 4. Methodology

To determine the cracking, yield and ultimate capacities of CFRP-reinforced concrete beams under pure torsion, five experimental specimens from Mohammadzadeh et al. are used [10]. Wavelet transform coefficients

obtained from the response analysis and perturbation time of the signal details of the beam end torsion angle are investigated and their compatibility with the time of the concrete cracking and yielding are discussed. Next, to the results of three experimental specimens from the reference [11] are utilized to determine the yield and ultimate capacities of CFRP-reinforced steel beams filled with concrete under the influence of bending. The wavelet transform coefficients and the perturbation time of the signal details of the vertical displacement in the middle of the beam are investigated corresponding to the steel yielding moment and the ultimate strength of the section.

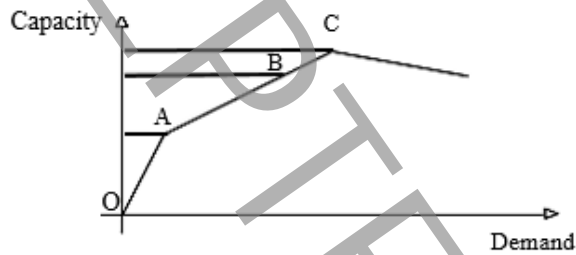
## 5. Results

In the present study, we first investigated the response of CFRP-reinforced experimental specimens under pure torsion, so that the torsion angle corresponding to the first cracking and yielding of the concrete is determined. The experimental results were analyzed by the discrete wavelet transform, and the damaged area was specified as fluctuations in the wavelet coefficient curve. It was declared that using this method, the time of collapse and the corresponding torsion angle could be determined at first crack and yielding of the concrete by the beam end torsion angle. By comparing the experimental results and the wavelet analysis, it can be seen that the torsional moment associated with the cracking angle obtained from the wavelet analysis corresponds to the torsional moment of the section in the laboratory results, which is obtained indirectly. Also, through numerical modeling of the steel beam filled with the CFRP-reinforced concrete in Abacus, the response of the vertical displacement of the middle of the beam was verified in the laboratory under the influence of bending. Then the response of the vertical displacement of the middle of the beam was analyzed using discrete wavelet transform and the perturbation area of the first-level details of the signal corresponding to the steel yield moment and the ultimate section moment were compared with the corresponding values from the experimental results. Therefore, with this method and the diagram of the response of the vertical displacement of the middle of the beam, the vertical displacement corresponding to the yielding of the steel and the ultimate section was determined. Studies show that the mother wavelet functions of sym7 and db4 have very good performance in detecting the cracking, steel yield and ultimate state in CFRP-reinforced beams and their corresponding moments. Therefore, the discrete wavelet transform has a high capability to analyze the experimental and numerical data responses and shows the discontinuities of the signal as perturbations on the wavelet coefficient graph. "Table 1" shows the results of cracking torsional moment, yielding torsional moment, and ultimate torsional moment of five experimental specimens under the influence of pure torsion obtained by experiments and the corresponding results obtained from wavelet transform, as well as the error percentage.

**Table 1. Cracking, yielding, and ultimate torsional moments of the beam obtained from experiments [10] as well as the wavelet transform results**

Experimental Specimens	Cracking Torsional Moment (Exp.) <sup>1</sup> (kN.m)	Cracking Torsional Moment (Wav. Trans.) <sup>2</sup> (kN.m)	Error. Percentage %	Yielding Torsional Moment (Exp.) (kN.m)	Yielding Torsional Moment (Wav. Trans.) (kN.m)	Error Percentage %	Ultimate Torsional Moment (Exp.) (kN.m)
ACW1	13.33	13.148	1.365	17.46	16.95	2.921	21.41
ACW2	15.15	15.176	0.17	17.63	18.01	2.155	25.26
BCW1	12.46	-	-	22.5	-	-	29.48
BCUJ	16.12	16.09	0.186	23	23.63	2.739	29.85
CCW1	16.56	16.47	0.543	28	28.23	0.821	33.87

<sup>1</sup> Exp.: Experimental, <sup>2</sup> Wav. Trans.: Wavelet Transform



**Figure 1. Behavioral diagram of reinforced concrete cross section under loading**

"Table 2" shows the results of the ultimate torsional moment of three specimens steel beams reinforced by CFRP and filled with the concrete under the influence of bending obtained by the numerical analysis and the corresponding results obtained from wavelet transform and error percentage.

**Table 2. Ultimate bending moment obtained from numerical analysis and ultimate bending moment obtained from wavelet transform**

Specimens	Ultimate Bending Moment (Num.) <sup>1</sup> (kN.m)	Ultimate Bending Moment (Wav. Trans.) <sup>2</sup> (kN.m)	Error Percentage %
FWB-L1(3)	28	28.45	1.61
FWB-L2(3)	34.54	34.1	1.273
FWB-L3(2)	28.7	29.2	1.742

<sup>1</sup> Num.: Numerical, <sup>2</sup> Wav. Trans.: wavelet Transform

## 6. Conclusion

According to the results, it can be shown that the mother wavelet functions of sym7 and db4 have very good performance in detecting the steel cracking, yielding and ultimate state in the CFRP-reinforced beams and their corresponding moments. Therefore, the discrete wavelet transform has a high capability to analyze the experimental and numerical data responses and shows the discontinuities of the signal as perturbations on the wavelet coefficient diagram.

## 7. References

- [1] Sumitoro, S., Matsui, Y., Kono, M., Okamoto, T., Fujii, K., 2001. "Long span bridge health monitoring system in Japan", *Health Monitoring and Management Systems*, Proceedings of SPIE 4337, 517–524.
- [2] Joo, D. J., 2012. "Damage Detection and System Identification using a Wavelet Energy Based Approach", (Doctoral dissertation, Columbia University).
- [3] Su, W. C., Le, T. Q., Huang, C. S., Lin, P. Y., 2018. "Locating damaged storeys in a structure based on its identified modal parameters in Cauchy wavelet domain", *Applied Mathematical Modelling*, 53, 1–19.
- [4] Zhong, S., Oyadiji, S. O., 2011. "Detection of cracks in simply-supported beams by continuous wavelet transform of reconstructed modal data", *Computers and Structures*, 89(1-2), 127–148.
- [5] Yang, C., Oyadiji, S. O., 2017. "Damage detection using modal frequency curve and squared residual wavelet coefficients-based damage indicator", *Mechanical Systems and Signal Processing*, 83, 385-405.
- [6] Patel, S. S., Chourasia, A. P., Panigrahi, S. K., Parashar, J., Parvez, N., Kumar, M., 2016. "Damage Identification of RC Structures using Wavelet Transformation", *Procedia Engineering*, 144, 336-342.
- [7] Kumar, P., Foufoula-Georgiou, E., 1997. "Wavelet analysis for geophysical applications". *Reviews of geophysics*, 35(4), 385-412.
- [8] Polikar, R., The wavelet tutorial, at <http://users.rowan.edu/~polikar/WAVELETS>.
- [9] Misiti, M., Misiti, Y., Oppenheim, G., Poggi, J. M. 1996. "Wavelet toolbox user's guide". *The Math Works Ins.* 2-36.
- [10] Mohammadzadeh, M.R., Fadaie, M., Rounagh, H., 2009. "Improving torsional behaviour of reinforced concrete beams strengthened with carbon fibre reinforced polymer composite".
- [11] Al Zand, A. W., Badaruzzaman, W. H. W., Mutalib, A. A., & Qahtan, A. H., 2015. "Finite element analysis of square CFST beam strengthened by CFRP composite material". *Thin-Walled Structures*, 96, 348-358.

ACCEPTED MANUSCRIPT