

Proposing a RC Fiber Frame Element Based on Local Stress Field Theory and Bar-Concrete Interaction

B. Yousefi¹, M. R. Esfahani^{1*}, M. R. Tavakolizadeh¹

¹ Department of Civil Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

ABSTRACT

This research presents an analytical model for developing a fiber frame element based on local stress field theory. The proposed formulation is developed through Lagrangian kinematics assumption to derive the weak form of the equations in large strain conditions. In this regard, the effect of bond-slip has been considered by removing perfect bond assumption. The governing equations for each element are developed by directional stiffness matrix in weak form. The extracted formula is based on the Timoshenko's beam theory, with axial, bending, and shear interaction effects in the domain of each element. The components of the stiffness matrix are defined through directional derivatives of the semi-linear form of the equations. Moreover, the suggested approach evolves from cubic Hermitian polynomials and the local stress field theory. The validation of the analytical method is provided by the available experimental tests. The implemented code could cover the overall behavior of reinforced concrete members, as well as, the maximum crack width, slip profile and the crack growth. The results show that such modeling method is capable of simulating RC members.

KEYWORDS

Fiber Frame Element, Smeared crack approach, Timoshenko's Theory, Bond-Slip Effect, Lagrangian Approach.

* Corresponding Author: Email: esfahani@um.ac.ir

1. Introduction

Fiber-based approaches are the most commonly used due to concurrently satisfying the accuracy, reliability, computational efficiency, and robust algorithmic performance, as well as the nonlinear flexural-shear interactions consideration [1, 2]. The state-of-the-art review on the existing frame models with the inclusion of shear responses was excellently presented in [3]. Although the literature presents plenty of other interesting formulation approaches based on force-based element [4-8] or displacement-based element [9-11], there is still a lack of methods that consider bond-slip behaviors based on a finite strain-description. Consequently, this paper is extended the displacement-based frame element of Limkatanyu and Spacone [12] which is different from those published previously. In other words, an exact multi-directional stiffness matrix is analytically derived based on the post-cracking bond-slip interaction between concrete and steel bars, as well as the presentation of a Timoshenko fiber frame model for large displacement analysis by using Green-Lagrange finite strain tensor. The present model is a simple applicable approach which is almost accurate and time saving, also it holds proper convergence compared to micro modeling methods.

2. Methodology

The approach can be applied to finite elements consisting of fiber beam-column elements which is prepared in MATLAB framework. The model has been constructed by using the equilibrium conditions of an infinitesimal segment with bond interfaces at bars and constant external forces.

In this research the weak formulation of updated Lagrangian (UL) kinematics is used to derive the finite element equations of a two-node Timoshenko plane beam element. The suggested approach evolves from cubic Hermitian polynomials, which has been well established by Bazoune et al. [13]. The main advantage of the developed expressions of shape functions over the classical shape functions is the shear deformation factors that can account for shear effects. Hence, the nonlinear strain vector ($\bar{\epsilon}$) including the axial strain (ϵ_{xx}) and the transverse shear strain (ϵ_{yx}) defined as:

$$\bar{\epsilon} = \begin{Bmatrix} \epsilon_{xx} \\ 2\epsilon_{yx} \end{Bmatrix} = \begin{Bmatrix} e - \gamma\kappa \\ \gamma \end{Bmatrix} \quad (1)$$

$$\begin{Bmatrix} e \\ \gamma \\ \kappa \end{Bmatrix} = \begin{Bmatrix} (1 + u'_x) \cos \theta + u'_y \sin \theta - 1 \\ -(1 + u'_x) \sin \theta + u'_y \cos \theta \\ \kappa \end{Bmatrix}$$

in which, the three strain quantities (e , γ , κ) characterize axial strains, shear strains, and curvatures, respectively. The bond-slip between the surrounding concrete and the i^{th} fiber of reinforcement (u_{slip}) and the normal strain of the concrete (ϵ_{xx}^{con}) and steel bar strain of i^{th} layer

(ϵ_{xx}^{bars}) are defined. Regarding the aforementioned aspects, the derivation of the weak form of the governing differential equations and its numerical implementation are written. In order to make finite element relations, a stable discretization is presented. As the local assembly proceed, continuous displacements field discretization is applied. Thus, the element stiffness matrix for continuum finite elements from the governing differential equation can be derived. Considering an incremental formulation of equilibrium, the tangent stiffness matrix is obtained through the first variation of the internal force vector in each degree of freedom direction which named multi-directional stiffness matrix as:

$$\begin{aligned} & \{\bar{u}_N\}_i^T \int_L \underbrace{\lambda_N \left(\frac{\kappa}{B_w \{\bar{u}_w\}} \right) B_N dx}_{k_{u_N}^{u_N}} \{\Delta \bar{u}_N\}_i \\ & \{\bar{u}_N\}_i^T \int_L \underbrace{-\lambda_N \frac{\kappa(1 + B_N \{\bar{u}_N\})}{\{B_w \bar{u}_w\}^2} B_w dx}_{k_{u_N}^{u_w}} \{\Delta \bar{u}_w\}_i \\ & \{\bar{u}_N\}_i^T \int_L \underbrace{\begin{bmatrix} \lambda_N^s & 0 \\ 0 & \lambda_N^s \end{bmatrix} B_{slip} dx}_{k_{u_N}^{u_{slip}}} \{\Delta \bar{u}_{slip}\}_i \\ & \{\bar{u}_w\}_i^T \int_L \underbrace{-\lambda_w \left(\frac{\kappa}{B_w \{\bar{u}_w\}} \right) B_N dx}_{k_{u_w}^{u_N}} \{\Delta \bar{u}_N\}_i \\ & \{\bar{u}_w\}_i^T \int_L \underbrace{\lambda_w \frac{\kappa(1 + B_N \{\bar{u}_N\})}{\{B_w \bar{u}_w\}^2} B_w dx}_{k_{u_w}^{u_w}} \{\Delta \bar{u}_w\}_i \\ & \{\bar{u}_w\}_i^T \int_L \underbrace{-\begin{bmatrix} \lambda_w^s & 0 \\ 0 & \lambda_w^s \end{bmatrix} B_{slip} dx}_{k_{u_w}^{u_{slip}}} \{\Delta \bar{u}_{slip}\}_i \\ & \{\bar{u}_{slip}\}_i^T \int_L \underbrace{B_{slip}^T \lambda_{slip}^{\sigma} \left(\frac{\kappa}{B_w \{\bar{u}_w\}} \right) B_N dx}_{k_{u_{slip}}^{u_N}} \{\Delta \bar{u}_N\}_i \\ & \{\bar{u}_{slip}\}_i^T \int_L \underbrace{-B_{slip}^T \lambda_{slip}^{\sigma} \left(\frac{\kappa(1 + B_N \{\bar{u}_N\})}{\{B_w \bar{u}_w\}^2} \right) B_w dx}_{k_{u_{slip}}^{u_w}} \{\Delta \bar{u}_w\}_i \\ & \{\bar{u}_{slip}\}_i^T \left(\int_L \underbrace{B_{slip}^T \lambda_{slip}^{\sigma} B_{slip} dx}_{k_{u_{slip}}^{u_{slip}}} + \int_L \underbrace{N_{slip}^T \lambda_{slip}^{\tau} N_{slip} dx}_{k_{u_{slip}}^{u_{slip}}} \right) \{\Delta \bar{u}_{slip}\}_i \end{aligned} \quad (2)$$

3. Constitutive models

Under uniaxial tension-compression or biaxial compressive stress state of the material, nonlinear material characteristics are considered (see [14], [15] and [16]). In the RC structures, the constitutive relation in the uncracked state is restricted to linear elasticity. The constitutive shear model is adapted according to Li [17] which was developed for modeling the nonlinear behavior of concrete elements. In this paper, the local stress field theory presented by Soltani, An and Maekawa [18] is considered.

4. Numerical Results and Solution

Some numerical examples are used to verify the accuracy and show the efficiency of the proposed material nonlinear frame element as well as the solution marching schemes. The iterative-incremental method (Arc-Length method) with a variable stiffness scheme was applied to analyze structures. Afterward, several numerical investigations were performed with the proposed model in order to study the effects of nonlinear shear deformations and flexural responses, simultaneously. The results of the nonlinear computer analyses are compared with the observed data and analytical results. Some of examples are considered as performed tests by Gilbert and Nejadi [19], Sasani, Werner and Kazemi [20], and Pham, Tan and Yu [21].

5. Conclusion

The main characteristics of the method are substantially the flexibility formulation and the constitutive relationship characterized by a fixed smeared crack model. The proposed model was calibrated and validated through a comparison with experimental results and various numerical analyses were performed in order to study the influence of nonlinear flexural-shear interaction. Thereafter, this method could yield accurate and convergent results in agreement with the problems.

6. References

- [1] M. Petrangeli, P.E. Pinto, V. Ciampi, Fiber element for cyclic bending and shear of RC structures. I: Theory, *Journal of Engineering Mechanics*, 125(9) (1999) 994-1001.
- [2] A. Saritas, F.C. Filippou, Inelastic axial-flexure-shear coupling in a mixed formulation beam finite element, *International Journal of Non-Linear Mechanics*, 44(8) (2009) 913-922.
- [3] P. Ceresa, L. Petrini, R. Pinho, Flexure-shear fiber beam-column elements for modeling frame structures under seismic loading—state of the art, *Journal of Earthquake Engineering*, 11(S1) (2007) 46-88.
- [4] E. Brunesi, R. Nascimbene, Extreme response of reinforced concrete buildings through fiber force-based finite element analysis, *Engineering Structures*, 69 (2014) 206-215.
- [5] E. Brunesi, R. Nascimbene, F. Parisi, N. Augenti, Progressive collapse fragility of reinforced concrete framed structures through incremental dynamic analysis, *Engineering Structures*, 104 (2015) 65-79.
- [6] D. Feng, C. Kolay, J.M. Ricles, J. Li, Collapse simulation of reinforced concrete frame structures, *The Structural Design of Tall and Special Buildings*, 25(12) (2016) 578-601.
- [7] X.H. Yu, D.G. Lu, K. Qian, B. Li, Uncertainty and sensitivity analysis of reinforced concrete frame structures subjected to column loss, *Journal of Performance of Constructed Facilities*, 31(1) (2016) 04016069.
- [8] E. Brunesi, F. Parisi, Progressive collapse fragility models of European reinforced concrete framed buildings based on pushdown analysis, *Engineering Structures*, 152 (2017) 579-596.
- [9] Z.-X. Li, Y. Gao, Q. Zhao, A 3D flexure-shear fiber element for modeling the seismic behavior of reinforced concrete columns, *Engineering Structures*, 117(Supplement C) (2016) 372-383.
- [10] R.S. Stramandinoli, H.L. La Rovere, FE model for nonlinear analysis of reinforced concrete beams considering shear deformation, *Engineering structures*, 35 (2012) 244-253.
- [11] M. Lezgy-Nazargah, An efficient materially nonlinear finite element model for reinforced concrete beams based on layered global-local kinematics, *Acta Mechanica*, 229(3) (2018) 1429-1449.
- [12] S. Limkatanyu, E. Spacone, Reinforced concrete frame element with bond interfaces. I: Displacement-based, force-based, and mixed formulations, *Journal of Structural Engineering*, 128(3) (2002) 346-355.
- [13] A. Bazoune, Y. Khulief, N. Stephen, Shape functions of three-dimensional Timoshenko beam element, *Journal of Sound and Vibration*, 259(2) (2003) 473-480.
- [14] K. Maekawa, H. Okamura, A. Pimanmas, *Non-linear mechanics of reinforced concrete*, Spon Press, 2003.
- [15] F.J. Vecchio, M.P. Collins, The modified compression-field theory for reinforced concrete elements subjected to shear, *Journal of the American Concrete Institute*, 83(2) (1986) 219-231.
- [16] Y. Zhuge, D. Thambiratnam, J. Corderoy, Nonlinear dynamic analysis of unreinforced masonry, *Journal of structural engineering*, 124(3) (1998) 270-277.
- [17] B. Li, Contact density model for stress transfer across cracks in concrete, *Journal of the Faculty of Engineering, the University of Tokyo*, (1) (1989) 9-52.
- [18] M. Soltani, X. An, K. Maekawa, Localized nonlinearity and size-dependent mechanics of in-plane RC element in shear, *Engineering structures*, 27(6) (2005) 891-908.
- [19] R.I. Gilbert, S. Nejadi, An experimental study of flexural cracking in reinforced concrete members under short term loads, University of New South Wales, School of Civil and Environmental Engineering, 2004.
- [20] M. Sasani, A. Werner, A. Kazemi, Bar fracture modeling in progressive collapse analysis of reinforced concrete structures, *Engineering Structures*, 33(2) (2011) 401-409.
- [21] A.T. Pham, K.H. Tan, J. Yu, Numerical investigations on static and dynamic responses of reinforced concrete sub-assemblages under progressive collapse, *Engineering Structures*, 149 (2017) 2-20.