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Mapped Moving Least Squares Approximation Used in Mixed Discrete Least Squares Meshfree Method

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ABSTRACT: The Mixed Least Squares Meshfree (MDLSM) method has shown its appropriate efficiency for solving Partial Differential Equations (PDEs) related to the engineering problems. The method is based on the minimizing the residual functional. The residual functional is defined as a summation of the weighted residuals on the governing PDEs and the boundaries. The Moving Least Squares (MLS) is usually applied in the MDLSM method for constructing the shape functions. Although the required consistency and compatibility for the approximation function are satisfied by the MLS, the method loses its appropriate efficiency when the nodal points cluster become too much. In the current study, the mentioned drawback is overcome using the novel approximation function called Mapped Moving Least Squares (MMLS). In this approach, the cluster of closed nodal was pointed maps to standard nodal distribution. Then the approximation function and its derivatives were computed incorporating some consideration. The efficiency of suggested MMLS for overcoming the drawback of MLS was evaluated by approximating the mathematical function. The obtained results showed the ability of suggested MMLS method to solve the drawback. The suggested approximation function was applied in MDLSM method, and used for solving the Burgers equations. Obtained results approved the efficiency of suggested method.

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1- Introduction

Over the last decade, the meshfree methods have received significant attention to solve the partial differential equations, governing the physical problems, due to their advantages compared to the existing mesh-based methods such as the Finite Element and the Finite Volume methods. In the meshfree methods, the discretization of the domain is implemented using a set of scattered nodal points instead of elements or cells used in the mesh-based methods. Hence, keeping any predefined connectivity among the nodes is not required in the meshfree methods unlike the mesh-based methods. Such property provides exclusive flexibility for the meshfree methods to overcome the drawback of the mesh-based methods, when dealing with moving boundary problems, and facilitate implementing the adaptive refinement approaches. Recently, sorts of meshfree methods has been presented to solve the variety of engineering problems. Smoothed particle hydrodynamic (SPH) and moving particle semi-implicit (MPS) are the well-known meshfree methods using the Kernel method to approximate the differential operators. Such methods successfully employed to simulate the fluid mechanics problems [1, 2]. Using the Kernel method in the SPH and MPS methods makes them simple and affordable; however, the Kernel method cannot guarantee the upper order of consistency for the approximated functions. Such restriction may lead special computational issues for the SPH and MPS. The series representation methods are the other class of approximations, used by a group of meshfree methods, which can provide the higher order consistency using raising the order of basis function. The element free Galerkin (EFG) and meshless local Petrov-Galerkin (MLPG) use the series representation method rather than Kernel method. Since the EFG and MLPG use the weak form of governing equations to discretize, they required a background mesh to implement the numerical integration procedure. Hence, the weak form type of meshfree methods was not considered as a truly meshfree methods. Unlike the EFG and MLPG, discrete least squares meshfree (DLSM) method uses a strong form of the governing differential equations so the method is purely independent of any mesh. Furthermore, the series representation type of approximation is used in the DLSM to avoid of the difficulties mentioned for the Kernel type of approximation used in SPH and MPS.

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More recently, the mixed formulation was used in the DLSM to improve the accuracy of the method. Suggested method is called Mixed Discrete Least Squares Meshfree (MDLSM) method. The method was successfully applied to solve the equilibrium [3] and propagation [4] problems. It was also developed to solve the Navier-Stokes equation [5].

The moving least squares (MLS) method is the well-known series representation type of approximations usually used by the EFG, MLPG, DLSM and MDLSM to construct the shape functions. Although the method is a strong approach for approximation, its efficiency decays where the nodal points are densely clustered. In the current study, the mapped moving least squares (MMLS) method was presented to overcome the mentioned drawback for the MLS. In this method, the clustered nodal points were mapped to the standard reference domain and the shape functions and their derivatives were computed in the standard patch, then the calculated information were returned to the main domain. The applied mapping procedure adjusted the distance between the nodal points at the standard reference patch to avoid of densely clustered point distribution leading to an accurate approximation for the MMLS compared to the MLS.

2- Mapped Moving Least Squares (MMLS) Approximation

The following Equation was used in the MLS method to approximate the unknown function, φ(X), as:

$$\varphi(X) = N^T(X)\hat{\varphi} \tag{1}$$

where φ represents the unknown nodal parameters and the shape function, N, is computes as:

$$N^T = P^T(X)F^{-1}(X)M(X) \tag{2}$$

Here:

$$F(X) = \sum_{j=1}^{num_s} w_j\left(X - X_j\right)P\left(X_j\right)P^T\left(X_j\right) \tag{3}$$

$$M(X) = \left[w_1(X - X_1)P(X_1),\ldots,w_{num_s}\left(X - X_{num_s}\right)P\left(X_{num_s}\right)\right] \tag{4}$$

The basis, P, and weight, w, functions are displayed as follows:

$$P^T = \left[1,x,y,x^2,xy,y^2,\ldots,x^n,\ldots,y^n\right]_{1\times s} \tag{5}$$

$$w_j(d) = \begin{cases} \frac{2}{3} - 4d^2 + 4d^3 & d \le \frac{1}{2} \\ \frac{4}{3} - 4d + 4d^2 - \frac{4}{3}d^3 & \frac{1}{2} \le d < 1 \\ 0 & d > 1 \end{cases} \tag{6}$$

The derivatives of shape functions can be computed by:

$$\frac{\partial N}{\partial x} = \frac{\partial P^T}{\partial x}F^{-1}M + P^T\frac{\partial F^{-1}}{\partial x}M + P^T F^{-1}\frac{\partial M}{\partial x} \tag{7}$$

Computing the derivatives may lose its proficiency dealing with densely clustered nodal point distributions, so, the suggested MMLS used the following strategy based on a mapping to avoid of such drawback. In this strategy, as shown in Figure 1, the nodal points, defined at the main domain, were mapped to a reference standard patch.

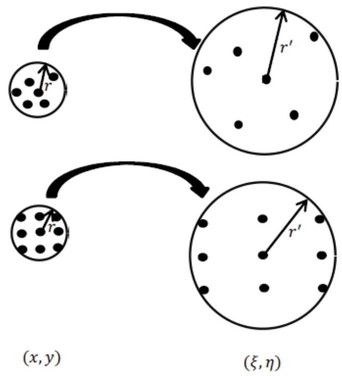


Figure 1. Schematic mapping strategy used in the MMLS

The following relation should be kept between the coordinates at the main domain, (x, y), and the standard reference domain, (ξ,η), as:

$$\xi = \frac{r'}{r}x, \eta = \frac{r'}{r}y \tag{8}$$

Using Eqs. 7 and 8, the shape function and its derivatives can be calculated as:

$$N(x,y) = N(\xi,\eta) \tag{9}$$

$$\frac{\partial N}{\partial x} = \frac{\partial N}{\partial \xi}\frac{d\xi}{dx} + \frac{\partial N}{\partial \eta}\frac{d\eta}{dx} \tag{10}$$

where

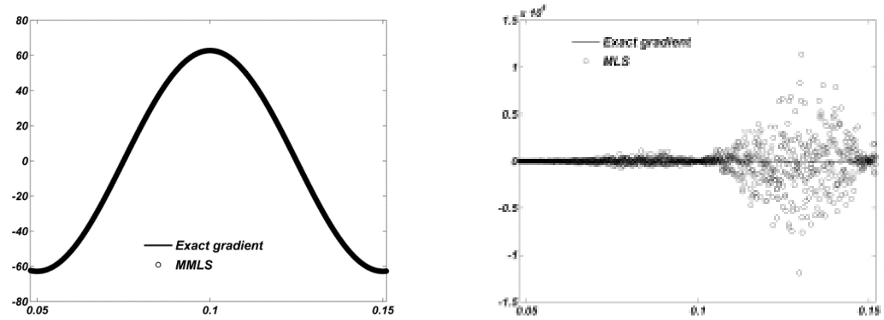
$$\frac{\partial N}{\partial x} = \frac{\partial N}{\partial \xi}\frac{r'}{r} \tag{10}$$

3- Results and Discussion

The approximation was implemented by the MLS and the MMLS using a densely uniform nodal distribution, dx=0.0001. The results were compared in Figure 2. Results proved that the suggested MMLS provided the more accurate approximation than MLS.

4- Conclusions

In this paper the MMLS approximation was proposed to overcome the drawback of existing MLS approximation when dealing with the densely clustered nodal distributions. The efficiency of suggested MMLS was evaluated using benchmark problems. The results showed the proficiency of the MMLS method to resolve the mentioned drawback of MLS.



(b) (a)

Figure 2. Comparing the results of MLS and MMLS

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