



## Statistical Quality Control Based on the Process Capability Index and Control Charts with Fuzzy Approach (Case Study: Water and Wastewater Company of West Azerbaijan Province)

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**ABSTRACT:** Statistical quality control is a method for monitoring the process to identify the underlying causes of changes and carrying out corrective actions. Process and capability control charts are two important applied tools for statistical quality control. In many actual systems in which accurate and certain information is not always available and the information is vague and fuzzy, fuzzy based methods can survey production process more precisely using appropriate linguistic terms and fuzzy numbers. In this study, fuzzy control charts were developed using fuzzy rules, and then the fuzzy actual capability index of process ( $C_{pm}$ ) was investigated in order to evaluate the precision, accuracy and performance of production process in the fuzzy state. The results of the studies performed on the quality of water flowmeters in the urban water and wastewater company of West Azerbaijan province showed that using fuzzy rules provides more decision-making options to decision-makers compared to the crisp data and provided more precise division about the product quality. Also, the fuzzy actual capability index of process could propose a more precise analysis of the process taking into account the average, target value and process variance, simultaneously. The values of the fuzzy actual capability index of process in the studied case were less than one, showing that the conditions of the production process are unfavorable.

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### 1- Introduction

Since humans gain the ability to production a product, it has also tried to control its quality. Quality control refers to the use of specific particularity of a product or service in order to design, production, maintenance and improve the quality of the product or service. Statistical process control has been widely used in quality control. It is a method for monitoring processes to identify specific causes of changes and reviser implementation proceedings. The control charts and process capability analysis are two applications of statistical quality control that perform an important role in improving the quality of processes and products. In many processes, Control charts have been used in classical situations where data and information are accurate and definite. These charts categorize the process into two categories under control and out of control using the exact data [1].

In many processes, statistical data is vague and fuzzy or information about the process is incomplete. In this cases fuzzy control charts are used. Fuzzy sets by defining membership functions and using vague data whit benefit from triangular or trapezoidal fuzzy numbers, provided different levels of decision for decision makers [2]. The

theory of fuzzy sets was utilized to add more information and flexibility to process analysis. Another control tool was the process capability. This index is a measure by which the process ability to produce products in accordance with customer satisfaction can be assessed [3]. Zade (1965) introduced fuzzy sets to classify subjective data [4]. Kaya & Kahraman (2011) Used the fuzzy measurement process for capability of analysis and fuzzy control charts in an industrial area in Turkey. In this research, they presented rules for surveying the status of samples. They also developed the  $C_p$  index for evaluating the production process [3]. Wooluru et al. (2014) studied the  $C_{pmk}$ ,  $C_{pm}$ ,  $C_p$  and  $C_{pk}$  indices and showed that the  $C_{pmk}$  index was more favorable than other indicators [5].

### 2- Methodology

To control a qualitative characteristic, the mean and its variance over time were examined. The average of the process was controlled by the mean control charts  $\bar{X}$ . The process variability can be controlled by control charts in standard deviations or range of changes with  $S$  and  $R$  charts.

#### 2- 1- Steps to design fuzzy control charts

The steps to design the fuzzy control charts are as

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follows:

Step 1: The samples were converted to fuzzy numbers. A quality attribute was assumed to be defined as an approximation of X that can be stated by a trapezoidal fuzzy number (a, b, c, d).

Step 2: The qualitative characteristic of the process was calculated and controlled by measuring the sample size and the mean was obtained as follows:

$$\bar{X} = \left( \frac{\sum_{i=1}^n a_i}{n}, \frac{\sum_{i=1}^n b_i}{n}, \frac{\sum_{i=1}^n c_i}{n}, \frac{\sum_{i=1}^n d_i}{n} \right) \quad (1)$$

$i=1, 2, 3, \dots, n$  ;  $j=1, 2, 3, \dots, m$

The standard deviation ( $\tilde{s}$ ) was also defined as follows:

$$\tilde{s}_j = \sqrt{\frac{\sum_i^n ((X_{a_i}, X_{b_i}, X_{c_i}, X_{d_i})_{ij} - (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d))^2}{n-1}} \quad (2)$$

Step 3: Based on the values obtained in the step 2, the upper control limits (UCL) and lower control limits (LCL) of the mean ( $\bar{X}$ ) were determined as follows, in which  $A_3$  is a constant value and determined according to the sample size [6].

$$U\tilde{C}\bar{L}_{\bar{X}} = C\tilde{L} + A_3\tilde{S}, \quad C\tilde{L}_{\bar{X}} = C\tilde{L}, \quad L\tilde{C}\bar{L}_{\bar{X}} = C\tilde{L} - A_3\tilde{S} \quad (3)$$

Also, the control range in the standard deviation ( $\tilde{s}$ ) was calculated as follows, in which  $B_3$  and  $B_4$  are constant values and determined according to the sample size.

$$U\tilde{C}L_S = B_4\tilde{S}, \quad C\tilde{L}_S = \tilde{S}, \quad L\tilde{C}L_S = B_3\tilde{S} \quad (4)$$

Step 4: In this step, the samples were placed in the control ranges and evaluated according to the following rules:

Rule 1: The calculated sample was within the control limits. In this case, the sample was controlled.

Rule 2: The sample was outside of the control limit. In this case, the sample was out of control.

Rule 3 and 4: In this case, a part of the sample was on the control limit that was considered to be in control or relatively in control.

Rule 5: In this case, a part of the sample was placed in the upper limit or another part was placed in the lower limit, that was considered to be in control or relatively in control [3].

Since the above mentioned rules were not covered all of the control process states, therefore in this research, the following rules were considered.

Rule 6: A part of the sample was located outside of the upper limit, that in case the samples were out of control or rather out of control.

Rule 7: A part of the sample was outside of the lower limit. In that case the samples were out of control or rather out of control.

## 2- 2- Fuzzy process capability indices

In the most of cases, it is not possible to determine the status of the process only by referring to the  $C_p$  index. The  $C_p$  index was not considered the location of the process average relative to the specifications' limits, but only measures the distance between the specifications' limits

relative to the distance [3]. Therefore, this index was not reflected the actual performance of the process. In this case, the  $C_{pm}$  will be a solution to the problem. This indicator is based on the square error idea and focuses on measuring the ability of the process to accumulate on the target, which shows the value on the target. To determine the accuracy and precision of a product, determine a product, this limit was consist of the upper specification limit (USL) and lower specification limit (LSL). These limits are defined as follows:

$$U\tilde{S}L = (u_a, u_b, u_c, u_d), \quad L\tilde{S}L = (l_a, l_b, l_c, l_d) \quad (5)$$

Now using the above specifications, the  $C_{pm}$  index can be defined in fuzzy mode as follows:

$$C_{pm} = \left( \frac{U\tilde{S}L - L\tilde{S}L}{6\tau} \right) = \left( \frac{u_a - l_d}{6\tau}, \frac{u_b - l_c}{6\tau}, \frac{u_c - l_b}{6\tau}, \frac{u_d - l_a}{6\tau} \right) \quad (6)$$

$$\tau^2 = E[(X - T)^2] = E[(X - \mu)^2] + (\mu - T)^2 = \sigma^2 + (\mu - T)^2 \quad (7)$$

Then:

$$\tau = \sqrt{\sigma^2 + (\mu - T)^2} \quad (8)$$

Where  $\sigma^2$  is the square of the standard deviation and is defined in fuzzy mode as follows:

$$\sigma^2 = \frac{R}{d_2} = (S_a, S_b, S_c, S_d) \quad (9)$$

in which,  $d_2$  is a constant value and is determined according to the sample size, and R is the range of changes. The T target value is also calculated using the upper and lower characteristics limits.

$$T = \frac{U\tilde{S}L + L\tilde{S}L}{2} = \left( \frac{u_a + l_d}{2}, \frac{u_b + l_c}{2}, \frac{u_c + l_b}{2}, \frac{u_d + l_a}{2} \right) \quad (10)$$

## 3- Results and Discussion

Water and wastewater Company of West Azarbaijan province was considered as a case study in this research. The data obtained from the laboratory of Water flow meters was converted into trapezoidal fuzzy numbers in three subgroups (normal, transfer and the least discharges). According to the obtained results, there are differences in the status of the flow meters in the three cases of using fuzzy rules, defuzzification with mean and standard deviation. In general, the case of using fuzzy rules is more sensitive to process changes, and decision making about the process status is more cautious. To get the process capability, samples that are out of control should be removed from the process. According to the values of  $C_{pm}$  ( $C_{pm} < 1$ ), process conditions are unfavorable. Therefore this process needs improvement to reduce the amount of process variations and deviations, and to increase the process capability.

## 4- Conclusion

In this study fuzzy rules were developed to survey the quality of equipment more accurately. In using fuzzy rules, the judgment was done based on the percentage of samples that fall within the in control or out of control ranges. Fuzzy rules provided other classifications for

decision makers and manufacturers in order to better decision making in comparison to crisp methods. In this paper, for evaluation of the accuracy of the production process performance, the  $C_{pm}$  actual process capability index was developed in fuzzy mode with the aim of measuring the efficiency of processes according to standard specifications. This index provided more information about the mean location of the process and also showed more sensitivity than standard deviation. It can be concluded that the  $C_{pm}$  index was the best indicator for determining the overall status of the process.

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