A Mixed Analytical Approach based on Semi-Timoshenko Planar Fiber Frame Element and Modified Compression Field Theory in RC Structures

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ABSTRACT: An accurate assessment of the behavior of structures by an analytical method should be able to estimate the initial stiffness of the structure, the maximum capacity and the local and global ductility. In this research, in order to simulate the nonlinear behavior of reinforced concrete structures under monotonic loading, a new fiber beam-column element was developed with a displacement control method using linearized arc-length approach. The formulation of the implemented element was based on the combination of Bernoulli and Timoshenko's theory along with the axial, flexural, and shear interaction effects of each element. The cross-sectional area of each element in Gaussian points was equivalent to a set of discrete fibers with uniaxial constitutive behavior in the process of nonlinear solution. Also, in order to consider the elemental shear deformation, the four-way smeared cracked approach and the modified compression field theory (MCFT) was considered in nonlinear shear analysis using the direct-displacement control algorithm in the main sub-program. The reference configuration of numerical formulation was considered according to the configuration of the previous step and the initial configuration, simultaneously. The analytic approach of the algorithm had the ability to change the updated Lagrangian formulation to the total Lagrangian in accordance with the problem-solving convergence. The developed fiber element was validated by numerous experimental experiments and the evaluation of the proposed analytical method was tested. The proposed method led to an appropriate solution and an acceptable convergence process with high processing speed for problems with mixed combinational mechanisms.

1- Introduction

Numerous efforts are done to improve upon the accuracy of conventional displacement-based beam–column finite elements for material nonlinear analysis of frame structures by many researchers. In primary simulation of beam-column elements with nonlinear behavior, nonlinear rotating springs is used at the lengths ends and critical points. Wen and Janssen's multiple component approach [1], Giberson's one component model [2] and Clough and Johnson's two components model [3] clearly exemplified these simulations.

The nonlinearity is treated using smeared plasticity approaches after discrete plasticity was introduced. In such approaches, nonlinear behavior is distributed in the cross sections of the elements. Kabeyasawa et al. [4] proposed a macro model called Three Vertical Line Element Model (TVLEM) for numerical modeling of reinforced concrete shear walls. The TVLEM consists of rigid zones at the top and the bottom of the story walls elevations, and two truss elements to represent axial stiffness. Subsequent development of TVLEM was Multiple-Vertical-Line

Element Model (MVLEM) presented by Vulcano et al. [5]. In this model, two truss elements were replaced with multiple vertical elements, and, based on researches of Kunnath et al. [6], and Spacone et al. [7], fiber elements were developed.

In this paper, updated Lagrangian approach was selected among main aspects of kinematic descriptions and an approach was improved to handle the problem of field inconsistency found in 2D Timoshenko planar frame elements for small strains-displacements with separate uniaxial fibers based on Orackal et al. [8]. Shear behavior of each element was covered based on the Modified Compression Field Theory (MCFT) which can be developed for general cases. In the proposed methodology, material nonlinearity was done through fiber arbitrary constitutive models and MCFT's approach was assigned to elements as shown in the following. The present model is a simple applicable approach which is almost accurate and time saving, also it holds proper convergence compared to micro modeling methods.

2- Element formulations

The approach can be applied to finite elements consisting of fiber beam-column elements which was prepared in

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MATLAB framework. The model has been constructed by combining the constitutive law for each fiber with a set of parallel fiber series. The main purpose of using fiber elements through a set of these series was improvement of seismic behavior of the proposed solution with employing shear constraint elements and consistent nonlinear behavior of materials (RC).

3- Kinematics of updated Lagrangian semi-Timoshenko fiber-based model

In this research, the general formulation of updated Lagrangian (UL) kinematics was used for derivation of the finite element equations of a two-node Timoshenko plane beam element. Formulations were implemented based on Timoshenko model with local first-order shear deformation effects. Also, theory could account for nonlinear behavior due to material nonlinearity due to constitutive behavior of each fiber.

Using the shape functions of the reference element, the discretization of the variables was performed. The elements with distributed nonlinearity were formulated with the classical stiffness method using cubic Hermitian polynomials presented by Bazoune and Khulief [9] to approximate the deformations along the element. According to Hermite interpolation, the degrees of freedom for each element were the displacements in orthogonal directions and slopes at the two nodes as Equations (1 & 2).

$$\theta(x)=[\varphi_{3,x}(x)\ \varphi_{4,x}(x)\ \varphi_{5,x}(x)\ \varphi_{6,x}(x)]*[v_1\theta_1v_2\theta_2]^T$$
$$\varphi_{4,x}(x)=1/(2\Phi_z)6(x/L)^2-2(3+\Phi_z)(x/L)+(1+\Phi_z)) \tag{1}$$

$$\varphi_{5,x}(x)=1/L\Phi_z\ (-6(x/L)^2+6(x/L)+(\Phi_z-1))$$
$$\varphi_{6,x}(x)=1/(2\Phi_z)(6(x/L)^2+2(\Phi_z-3)(x/L)+(1-\Phi_z)) \tag{2}$$

Where Φ_z is presented as shear slenderness which defined as Equation (3):

$$\Phi_z=1+(12EI_z)/(\kappa_y GAL^2) \tag{3}$$

Where $EI_z=\sum_{nf=1}^{ntf\ (x)}E_{nf}\ y_{nf}^2$ and κ_y are defined as shear stress correction factor which assumed 0.833 for rectangular sections. The variable G is shear modulus obtained by a subroutine program based on MCFT direct analysis which introduced by Vecchio and Colins [10]. In the following, section deformations are related to nodal displacements:

$$\tag{4}$$

The section resistance forces are computed from the fiber stress distribution and section stiffness ($K_s\ (x_i)$) is assembled from the fiber stiffnesses. This force is implemented according to Equation (4), and the following stages lead to the calculation of element stiffness as Equation (5):

$$\tag{5}$$

4- Constitutive models

Under uniaxial tension-compression or biaxial compressive stress state of the material, nonlinear material characteristics were considered (see [11], [10] and [12]). In the RC structures, the constitutive relation in the uncracked state was restricted to linear elasticity. The constitutive shear model was adapted according to Li [13], which was developed for modeling the nonlinear behavior of concrete elements. In this paper, Modified Compression Field Theory (MCFT) based on fixed smeared crack approach was implemented in the subroutine of the main program.

5- Numerical Implementation and Solution

Some numerical examples were used to verify the accuracy and show the efficiency of the proposed material nonlinear frame element as well as the solution marching schemes. The iterative-incremental method (Arc-Length method) with a variable stiffness scheme was applied to analyze structures. Afterward, several numerical investigations were performed with the proposed model in order to study the effects of nonlinear shear deformations and flexural responses, simultaneously. The results of the nonlinear computer analyses were compared with the observed data and analytical results. Some of examples were considered as Vecchio and Palermo' shear wall test [14], Chun and Kim's RC connection test [15] and two story reinforced concrete frame model was tested by Vecchio and Emara [16].

6- Conclusions

The main characteristics of the method were substantially the flexibility formulation and the constitutive relationship characterized by a fixed smeared crack model. The proposed model was calibrated and validated through a comparison with experimental results and various numerical analyses were performed in order to study the influence of nonlinear flexural-shear interaction. Thereafter, this method could yield accurate and convergent results in agreement with the problems.

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