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Project Risk Analysis Using an Integrated Probabilistic Beta-S Model and Multi-Parameter Copula Function

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ABSTRACT: One of the key attributes of any project is its time and cost constraints. Nowadays project-oriented organizations are looking for more advanced, accurate and efficient methods rather than traditional project management practices. Earned Value Management (EVM) is a well-accepted methodology to combine time, cost and scope of the projects reported in terms of Cost Performance Index (CPI) and Schedule Performance Index (SPI). However, a deterministic EVM cannot consider uncertainties of time and cost of activities of project and the correlation structure amongst them, which are inevitable and prevalent in any project. Therefore, an advanced probabilistic EVM is needed. A literature review reveals that there are only very limited studies in this area with different levels of complexity, maturity and limitations. In this study after defining the probability Number density functions, i.e., pdfs, of time and cost of every activity of project's scheduled plan and their correlation structure, using Primavera Risk Analysis®, as a commercially available project risk analysis software, and Monte Carlo Simulation (MCS), in every iteration a time based cumulative cost of project, the so-called S-curve is created and normalized to be the inputs for curve fitting into a four parameter Beta-S function. Hence, for every iteration the corresponding values of these parameters can be calculated and the best performing marginal pdfs, be derived. In this paper in a novel approach, copula functions are employed to bind together these pdfs with a high level of efficiency and accuracy in terms of reliving the limitation of their belongingness to the same parametric group of marginal pdfs, e.g., multivariate Gaussian joint probability distribution. The proposed model collates all propagated uncertainties of the project activities in a single probabilistic closed form function. This copula function can be used in estimation of the performance indices of a probabilistic EVM and more importantly fed into a Bayesian updating scheme to estimate the project future performance more accurately.

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1-Introduction

From the earliest projects' stage of cost estimation which results in Tender price to the latest stage of "Closing", stakeholders and sponsors of projects tend to predict cost and time status of projects to avoid any unpredictable risk which may jeopardize benefit of the project. Hence, it is vital to be well aware and prepared for fluctuations in environment of projects.

One of the methods, which holds a considerable reputation amongst similar methods of time-cost prediction of projects, is EVM (Earned Value Management). EVM is also such a standard and widely used method that has proved its efficiency in Mega projects. The first limitation of EVM is the deterministic nature of the CPI[1] and SPI[2] parameters whereas the stochastic behavior of project. Another challenge is presenting Schedule Variance via monetary units. It means that EVM introduces deviations in time by SPI parameters which consists of cost units [1].

$$SPI = \frac{EV}{PV} \tag{1}$$

1. Cost Performance Index
2. Schedule Performance Index

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Moreover, EVM can not make reliable results in early stages of project. Therefore, Lipke (2002b) developed statistical approach to EVM to predict total cost of project [2]. Vandevoorde and Vanhoucke (2006) developed three different methods of time-cost forecasting and stated that EVM is a more reliable method among them [3].

In recent years, using cumulative curve of project progress, which is called S-Curve, has appealed attention of researchers in the field of Earned Value Management. S-Curve has got this capacity to be used as a unique ID of the entity of project. Pajares and López-Paredes (2011) used Monte Carlo simulation to obtain the statistical distribution of the cost and the duration at the end of the project. This information is used to select the confidence level (both in terms of time and in terms of cost) that will be used to monitor the performance of the project [4].

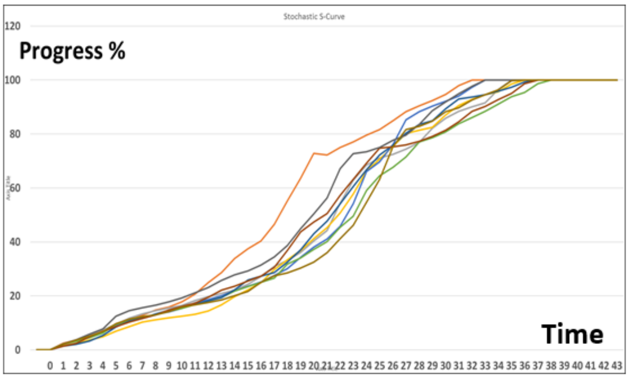
It is also possible to combine S curve with stochastic nature of time-cost of the project. Barraza et al. introduced concept of stochastic S curves (SS curves) to determine forecasted project estimates as an alternative to using deterministic S curves and traditional forecasting methods [5].

2-Beta-S Model

By assigning probability distributions to any of quantities of time and cost of each activity of project, and combining Monte Carlo simulation and CPM1 method to compute total cost and time of project, a network of time and cost quantities will be resulted (Figure 1). Any of these random variables could be assumed as a Probability Distribution Function (PDF), which are called marginal distribution functions of T, C.

To determine status of time-cost random variables of project during the whole duration of project, Kim and Reinschmidt (2007) presented a stochastic model, based on Beta distribution. They assumed that each S curve is visually similar to a beta cumulative distribution and so could be stated and fitted to a beta form [6].

Figure 1. Stochastic S-Curves of Project



Each Beta distribution has got 4 parameters A, B, α, β. In Beta-S model, A=0 and B=T which T is equal to the total time of project in any iteration.

(2)
$$f(x;\alpha.\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{(x-A)^{\alpha-1}\,(x-B)^{\beta-1}}{(B-A)^{\alpha+\beta-1}}$$
$$\alpha.\beta > 0 \,.\, A \le x \le B$$

After simulation of time-cost of project and fitting each S curve by Beta distribution, 10000 outputs will be obtained. To determine marginal distribution of any of parameters α, β, C, T it is expected to perform Goodness of Fit Test (GoF) to make sure the proposed probability distribution is the best fit to the empirical data.

In this paper a project consisting of 50 activities is considered and simulated 10000 times, so the marginal distributions of α, β, C, T are presented in the following Equations 3a-d.

(3-a) α ~ Lognormal (0/6627,0/1577)
(3-b) β ~ Lognormal (2/936,0/2189)
(3-c) C ~ Normal (2/5179,0/0490)
(3-d) T ~ Normal (10/2865,0/4395)

1. Critical Path Method

3-Copula Function

The copula function along with the marginal functions provides the possibility of modeling the random events for researchers. In fact, copula is a multivariate distribution the marginal distributions of which are distributed uniformly over [0,1] [7].

In this paper, a widely used elliptical copula, t-copula is used to bind basic parameters of project. For n correlated variables (u1,…, un) with correlation matrix of ρ, ν degree of freedom multivariate t-copula is defined as follows

(4)
$$\mathrm{CopPdf}^{t}_{v,\rho}\,(u_1,\ldots,u_n;\rho) = |\rho|^{-\frac{1}{2}} \frac{\Gamma\left(\frac{\upsilon+N}{2}\right)\left[\Gamma\left(\frac{\upsilon}{2}\right)\right]^{N}\left(1+\frac{1}{\upsilon}\varsigma^{T}\rho^{-1}\varsigma\right)^{-\frac{\upsilon+N}{2}}}{\Gamma\left(\frac{\upsilon+1}{2}\right)\Gamma\left(\frac{\upsilon}{2}\right)\prod_{n=1}^{N}\left(1+\frac{\varsigma_n^{2}}{\upsilon}\right)^{-\frac{\upsilon+1}{2}}}$$

Where $\varsigma_n = t_{\upsilon}^{-1}(u_N)$

Using the correlation matrix and considering the multivariate model, we can use copula t to determine the joint distribution function. In this paper, the software of MATLAB® is used to calculate proper multivariate Copula function of marginal distributions, so t-Copula is best choice in this case. The value of the joint distribution function is equal to the product of the values of the marginal functions α, β, C and T in a simulation iteration multiplied by the value of the copula function t in the same iteration (Eq.3). It is noteworthy that the number of outputs of the marginal functions is obtained from the simulation results and then separating the values present in the plausible area.

(5)
$$F_{Prior} = C \times T \times \alpha \times \beta \times Cop_{Pdf}(\alpha.\beta.C.T)$$

4-Bayesian Inference

Earned Value Management (EVM) has proven itself to be one of the most effective performance measurement and feedback tools for managing projects. It enables managers to close the loop in the plan-do-check-act management cycle. EVM has been called"management with the lights on" because it can help clearly and objectively illuminate where a project is and where it is going—as compared to where it was supposed to be and where it was supposed to be going. EVM uses the fundamental principle that patterns and trends in the past can be good predictors of the future [8].

Main Indices of EVM are CPI and SPI, which are identified as follows:

(6)
$$SPI = \frac{EV}{PV}\ ;\ CPI = \frac{EV}{AC}$$

In common EVM, CPI and SPI are both deterministic variables but in this paper, due to stochastic nature of time and cost of project, they turn into random variables.

$$SPI = \frac{EV}{PV}\ ;\ CPI = \frac{EV}{AC}$$

5-Results

Random variables of time-cost of project are statistically dependent and it is possible to model this dependency. One of the benefit of the model which is illustrated in this paper is that it integrates all uncertainties in any basic parameters of time and cost of project and quantifies total impact of them. Unlike previous researches, in this model, marginal distribution of time and cost of project could be fitted with any proper distribution after deploying GoF test and by introducing joint probability distribution of α, β, C and T, Bayesian updating comes to reach.

Authors of this paper have hope to carry on this field of study by deploying Bayesian Updating and benefit the merit of sophisticated methods to make the results more reliable.

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