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Experimental and FEM Study on Damaged Granitic Rock Using Second Rank Crack Tensor

K. Panaghi, A. A. Golshan*

Soil and Foundation Group, Department of Civil and Environmental Engineering, Tarbiat Modares University, Tehran, Iran

ABSTRACT: Any investigative approach towards rock behavior will necessitate inherent deficiencies such as pores and cracks to be taken into consideration. One of the methodologies employed to study cracked rock is to consider an equivalent continuum as for the domain with defects which will lend flexibility to experimental and numerical schemes due to its seamless effects on the constitutive relationships, hence reducing computational costs as well as experimental restraints in the laboratory. A case in point in such approach is the crack tensor model which is based upon the idea to represent cracks' size, orientation, and number density as one single entity through which proper geometric characterization of the in-situ rock is carried out. Following the introduction of crack tensor concept and its application in the technical literature, the current work focuses on the determination of second rank crack tensor using P-wave velocity measurements on damaged granite. The benefit of such approach is emphasized via its role in boosting the degree of accuracy of the numerical analysis code developed in Matlab that implements different compliance matrices for four different stages of loading. The calculation results showed promising trends in agreement with those of the experimental data. Apparently, more experimental procedure is required to improve results' accuracy in projects for which fulfilling more stringent regulatory requirements is a must.

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1. Introduction

The higher demands regarding construction of modern structures in today's industrial section has made the significance of efficient design considerations more prominent. Such motive will require more diligent evaluations be carried out which will usually render costly computations. Thereby, any attempt to reduce such cumbersome calculations would serve as highly appealing to practical engineering endeavors. One of the challenging aspects of such studies in rock mechanics analyses is to represent cracks in otherwise continuous domains. Accordingly, advanced numerical schemes have been developed within the academic society to address such requirements. The more elaborate numerical simulations, albeit valuable to some extent, are hindered by the computational costs that in some cases require bleeding-edge technology. Therefore, any alternative guaranteeing less timely procedures yet having enough precision is highly appreciated.

One of the interesting methods to characterize discontinuities within rock domains is via a mathematical entity referred to as "crack tensor". Such tensor would account for crack's size, orientation, and distribution in the medium [1]. This allows for further investigations regarding the mechanical behavior of the structures such as tunnels, caverns, and geological repositories constructed within rocky environments. In the following, a brief theoretical background in conjunction with the methodology employed herein as well as some discussion on the results are presented.

2.Representing Crack Geometry

The main attributes of defects in every domain can be categorized as their:

- Size
- Orientation
- Density

employing statistical analysis along with experimental work, a tensor taking the characteristics of cracks into consideration was introduced [1], the mathematical form of which is:

$$F = \frac{\pi\rho}{4}\int_{0}^{r_m}\int_{\Omega} r^3 n \otimes n \ldots \otimes n E(n,r)\, d\Omega dr \tag{1}$$

In the above, F represents crack tensor while r, $\mathbf{n}$, and ρ stand for crack diameter, orientation, and density, respectively. $E(\mathbf{n},r)$ is a function that accounts for crack density within the domain introduced by the solid angle Ω. Further research into the implementation of this approach lead to the constitutive relationship based on the crack tensor theory [2]:

$$\begin{bmatrix}\bar{\varepsilon}_{11}\\ \bar{\varepsilon}_{22}\\ \bar{\varepsilon}_{33}\\ \bar{\varepsilon}_{23}\\ \bar{\varepsilon}_{31}\\ \bar{\varepsilon}_{12}\end{bmatrix} = \frac{1}{D}\begin{bmatrix} F_{11}+\frac{D}{E} & -\frac{D}{E}\nu & -\frac{D}{E}\nu & 0 & \frac{1}{2}F_{13} & \frac{1}{2}F_{12}\\ & F_{22}+\frac{D}{E} & -\frac{D}{E}\nu & \frac{1}{2}F_{23} & 0 & \frac{1}{2}F_{12}\\ & & F_{33}+\frac{D}{E} & \frac{1}{2}F_{23} & \frac{1}{2}F_{31} & 0\\ & & & \frac{F_{22}+F_{33}}{4}+\frac{D}{4G} & \frac{1}{4}F_{12} & \frac{1}{4}F_{31}\\ & Sym. & & & \frac{F_{33}+F_{11}}{4}+\frac{D}{4G} & \frac{1}{4}F_{23}\\ & & & & & \frac{F_{11}+F_{22}}{4}+\frac{D}{4G}\end{bmatrix}\begin{bmatrix}\bar{\sigma}_{11}\\ \bar{\sigma}_{22}\\ \bar{\sigma}_{33}\\ 2\bar{\sigma}_{23}\\ 2\bar{\sigma}_{31}\\ 2\bar{\sigma}_{12}\end{bmatrix} \tag{2}$$

Corresponding Author: Email: golshani@modares.ac.ir in which ε, and s are strain and stress quantities, and D equals $2E/\pi$ with E as Young's modulus. One drawback in the employment of crack tensor theory is the tedious field measurements in the procedure for obtaining the tensor arrays. Corollary, an alternative approach based on the relationship between crack tensor and wave velocity tensor (V_{ij}) was introduced which facilitated related cracked domain studies [3]:

$$F_{ij} = k\left(V_{ij}^{-1} - \delta_{ij}\right), i = 1,2,3 \quad (3)$$

In the above formulation, δ_{ij} is Kronecker delta and κ is a coefficient depending on crack geometry.

3. Experimental Procedure and Results

Cuboid specimens of Inada granite with a height and side lengths of 74 mm and 34 mm were used to obtain the corresponding crack tensors using Equation 3. A total number of four specimens were employed to obtain the compliance matrices' arrays for different stages of the granite under loading. Having the confining pressures fixed at 10 MPa and 20 MPa, the specimens were loaded up to 90, 95, and 97 percent of their failure strengths while one specimen was kept intact. Wave velocity measurements were carried out in nine different directions to obtain six arrays of the second rank symmetric crack tensors. Table 1 illustrates the results of the wave velocity measurements obtained in this stage of the study..

Table 1.Wave velocities for the specimens in 9 directions

Direction	1	2	3	4	5	6	7	8	9
IGr34	2.92	3.42	3.75	3.39	3.33	3.13	3.22	3.68	3.71
IGr35	3.38	3.93	3.94	2.84	3.04	3.90	3.91	3.84	3.87
IGr39	2.79	3.04	3.75	2.68	2.68	3.80	3.95	3.61	3.71
IGr31	3.17	3.59	4.12	2.97	2.97	3.85	3.87	3.77	3.98

Following laboratory measurements, the crack tensors for each stage of damage were obtained using Equation 3 which were later used in the constitutive relationship defined in Equation 2. It is worth mentioning that the defined stress-strain relationship based on the crack tensor theory was incorporated in the numerical simulation procedure to compare the degree of accuracy of the predicted results with those of the experimental approach.

Table 2. 2nd rank crack tensors for Inada granite specimens

Specimen	Load Intensity	Crack Tensor
IGr34	**0**	$\begin{bmatrix} 1.743853 & -0.07429 & -0.06943 \\ & 1.25923 & -0.08242 \\ \text{Sym.} & & 0.983258 \end{bmatrix}$
IGr35	**90**	$\begin{bmatrix} 2.071187 & 0.095486 & 0.100752 \\ & 1.353073 & 0.105655 \\ \text{Sym.} & & 1.313442 \end{bmatrix}$
IGr39	**95**	$\begin{bmatrix} 2.754094 & 0.047463 & -0.23765 \\ & 1.875305 & 0.076015 \\ \mathit{Sym.} & & 1.044917 \end{bmatrix}$
IGr31	**97**	$\begin{bmatrix} 2.196944 & 0.032098 & 0.117804 \\ & 1.524077 & -0.05848 \\ \text{Sym.} & & 1.144891 \end{bmatrix}$

4. Numerical Simulation Procedure and Results

Quadratic tetrahedral elements (Figure 1) were used to model the behavior of the granitic rock using an FEM code developed in Matlab. The crack tensors obtained via the earlier stage of the study were used to represent the load intensities of 0-90, 90-95, 95-97, and 97-100 percent of the granite's failure strength. Figure 2 illustrates the results for both experimental and numerical procedures in the study. It is observed that the results indicate acceptable agreement within the practical engineering framework.

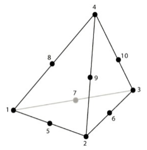

Figure 1. Tetrahedral element used in the numerical study

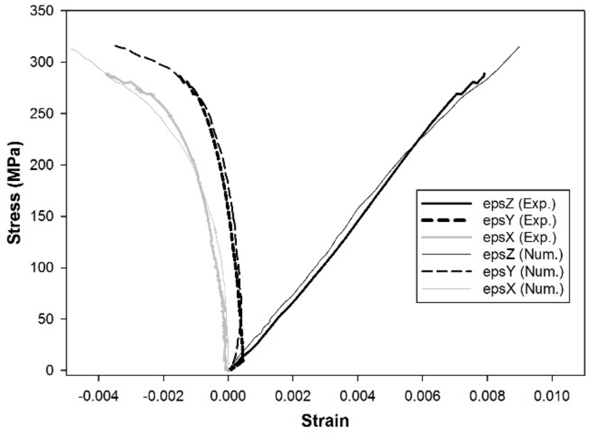

Figure 2. Comparison of the numerical and experimental results for the specimen with load intensity of 97 percent

5. Conclusions

The current study aimed at representing the usage of an efficient tool already employed in hydro-mechanical characterization of cracked domains in the numerical simulation of such media. Unlike the earlier studies that mainly relied on the field data-measurements, the current investigation adopted for the use of data derived from laboratory experiments which could potentially lessen the degree of effort in such analyses. The obtained results were subsequently introduced in the program which was coded based on the crack tensor-based behavioral law. The outcome showed that the current scheme could effectively be considered as a substitute for costly numerical simulations that in some cases might not result in meaningful rigor for practical purposes. Apparently, experimental results on a higher number of rock specimens would improve the numerical results' precision where required.

The uncertainties in the study, aside from the limited number of specimens, could be attributed to the measurement errors as well as the simplifying assumptions in the theoretical analysis such as considering all the cracks to be of the same shape. In addition, the degree of rigor in similar studies can be increased in case higher orders of crack tensor are employed. Not only do such ranks of the tensor would boost the accuracy of the results concerning domain's deformation, they could also be beneficial in studies related to the permeability and flow within the cracked rock. However, acquiring higher ranks of crack tensor would necessitate more elaborate procedures, the feasibility of which is under investigation by authors.

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References

[1] Oda, M., 1982. "Fabric tensor for discontinuous geological materials". Soil Found., 22(4), pp. 96-108.

[2] Oda, M., Suzuki, K., and Maeshibu, T., 1984. "Elastic compliance for rock-like materials with random cracks". Soil Found., 24(3), pp. 27-40.

[3] Takemura, T., and Oda, M., 2005. "Changes in crack density and wave velocity in association with crack growth in triaxial tests of Inada granite". Journal of Geophysical Research: Solid Earth, 110(B5).

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