



## A New Simplified Method of Time-dependent Column Shortening Analysis in Concrete Moment Frames

M. Jalilzadeh Afshari\*, A. Kheyroddin

Faculty of Civil Engineering, Semnan University, Semnan, Iran

**ABSTRACT:** Excessive column shortening in conventional one-step analyses is one of the most important aftereffects of ignoring the real behavior of concrete columns. This real behavior could only be achieved by staged application of gravity loads considering the long-term inelastic strains of concrete. Thereupon, consideration of adaptation between high-rise reinforced concrete structures' design and practical stages of construction has always been staunchly held forth by researchers. Neglecting the fact above may lead to serious incorrect outcomes of analyses especially in high-rise structures. Some of these adverse structural effects are extra induced bending moments in beams, expansion of progressive cracks in second or non-structural elements, and wasting the intended capacity for structural elements in the design stage. This paper deals with comprehensive nonlinear staged analyses of structures with various geometrical specifications and represents simple empirical equations to evaluate column shortening caused by creep, shrinkage, and time changes of modulus of elasticity in such a way that the proposed relations could be independent of conventional variables of CEB-FIP code. Results of validation process show high conformity of all proposed equations for up to 30 floors and also demonstrate the accuracy of proposed shrinkage relation even for the structures higher than the aforementioned limit.

### Review History:

Received: 6 March 2017  
Revised: 18 August 2017  
Accepted: 23 October 2017  
Available Online: 1 November 2017

### Keywords:

Nonlinear Sequential Analysis  
Conventional Analysis  
Column Shortening  
Creep  
Shrinkage

### 1- Introduction

This paper deals with the estimation of column shortening in special moment resisting frame systems using simple empirical equations which are independent of conventional geometrical parameters of common standard methods of estimating the axial time-dependent strains of concrete such as notional size. Besides the accurate and consistent estimation of shortening due to creep, shrinkage and time changes of characteristic compressive strength and the modulus of elasticity, compared to corresponding values of accurate nonlinear staged analysis [1-3], the equations proposed in the present research cover a wide range of special moment resisting frame structures with different heights and geometrical specifications and will be valid only for structures which have been designed optimally based on ACI318-14 regulations and internal forces caused by conventional one-step structural analysis without consideration of long-term effects and time-dependent strains of concrete.

### 2- Results and Discussion

The method proposed in the present study is capable of estimating the long-term behavior of structure in form of creep, shrinkage and elastic shortenings of each column of structure on the 1000th day of construction (as the time indicator when inelastic strains of concrete have almost reached their final value) in accordance with CEB-FIP 1990 regulations and without any need for performing the nonlinear staged analysis and definition of corresponding effective parameters. The aforementioned purpose is achieved just by the use of proposed equations which are only a function of height of column ( $h$ ), length of span ( $l$ ), number of floors ( $n$ ) and the number of intended floor ( $i$ ), as follows;

$$CS_{Creep}(i, h_i, n, l) = (Ai^3 + Bi^2 + Ci + D)G \exp(H.i)(h_i / 3.5) \quad (1)$$

$$CS_{Elastic}(i, h_i, n, l) = (Ai^3 + Bi^2 + Ci + D)G \exp(H.i)(h_i / 3.5) \quad (2)$$

$$CS_{Shrinkage}(i, h_i, n, l) = (Ei^F)(G.Ln(i) + H)(h_i / 3.5) \quad (3)$$

In which;

Corresponding author, E-mail: jalilzadeh.afshari@semnan.ac.ir

$$\begin{cases}
 A = a_1 n^2 + b_1 n + c_1 \\
 B = d_1 n^2 + e_1 n + f_1 \\
 C = g_1 n^2 + h_1 n + i_1 \\
 D = j_1 n^2 + k_1 n + l_1 \\
 E = m_1 \exp(n_1 n) \\
 F = o_1 n^{p_1} \\
 G = (a_2 l^2 + b_2 l + c_2) n^2 + (d_2 l^2 + e_2 l + f_2) n + (g_2 l^2 + h_2 l + i_2) \\
 H = (j_2 l^2 + k_2 l + l_2) n^2 + (m_2 l^2 + n_2 l + o_2) n + (p_2 l^2 + q_2 l + r_2)
 \end{cases} \quad (4)$$

Variables in above equations are floor number (i), span length (l), height of i-th floor (hi) and number of floors in structure (n) and other parameters are constant coefficients with values listed in Table 1 and Table 2 respectively for columns of C1 & C2 and C3-I & C3-II types.

### 3- Conclusions

Simplicity of proposed equations due to their dependent only to geometric characteristics including dimensions of span, height and number of floors and its independence from

the conventional parameters of shortening estimation in CEB-FIP 1990 regulations are among the benefits of method used in the present research. Results obtained from the evaluations in this research showed that:

- All proposed equations have validated for all special concrete moment frames with up to 30 floors with optimal design based on conventional one-step analyses in accordance with ACI318-14 regulations and initial assumptions of the article. The proposed equations can be used instead of time consuming staged analyses for estimation of shortening in different types of columns.
- The proposed equation in the present study in terms of shortening caused by shrinkage is valid even for structures taller than 30 floors and is able to estimate the effect of column's shrinkage with an acceptable error percentage.
- The presented shrinkage constants are almost equal for every type of column. Hence, the proposed equation for shrinkage is independent from situation and tributary area of column and each one of specific proposed series of shrinkage constants can be used to estimate the shrinkage shortenings of all types of columns with a very good approximation.

**Table 1. Constant coefficients of equations (1 to 4) for C1 and C2 column**

Constant	C1			C2		
	Creep	Elastic	Shrinkage	Creep	Elastic	Shrinkage
a <sub>1</sub>	1.9498×10 <sup>-9</sup>	1.7132×10 <sup>-9</sup>	-	2.3689×10 <sup>-9</sup>	2.3315×10 <sup>-9</sup>	-
b <sub>1</sub>	-8.4736×10 <sup>-8</sup>	-8.9331×10 <sup>-8</sup>	-	-9.1821×10 <sup>-8</sup>	-1.1716×10 <sup>-7</sup>	-
c <sub>1</sub>	2.9289×10 <sup>-7</sup>	9.8222×10 <sup>-7</sup>	-	-1.3320×10 <sup>-8</sup>	1.1842×10 <sup>-6</sup>	-
d <sub>1</sub>	-7.2789×10 <sup>-8</sup>	-3.2629×10 <sup>-8</sup>	-	-1.1974×10 <sup>-7</sup>	-5.6682×10 <sup>-8</sup>	-
e <sub>1</sub>	4.5947×10 <sup>-6</sup>	2.2522×10 <sup>-6</sup>	-	7.2076×10 <sup>-6</sup>	3.7967×10 <sup>-6</sup>	-
f <sub>1</sub>	-7.0731×10 <sup>-5</sup>	-4.6870×10 <sup>-5</sup>	-	-1.0457×10 <sup>-4</sup>	-7.2416×10 <sup>-5</sup>	-
g <sub>1</sub>	-4.2560×10 <sup>-7</sup>	-1.9345×10 <sup>-7</sup>	-	-6.1600×10 <sup>-7</sup>	-3.7090×10 <sup>-7</sup>	-
h <sub>1</sub>	2.1812×10 <sup>-5</sup>	1.4806×10 <sup>-5</sup>	-	2.2666×10 <sup>-5</sup>	1.9951×10 <sup>-5</sup>	-
i <sub>1</sub>	3.8596×10 <sup>-4</sup>	1.4281×10 <sup>-4</sup>	-	6.6420×10 <sup>-4</sup>	2.4838×10 <sup>-4</sup>	-
j <sub>1</sub>	3.8916×10 <sup>-7</sup>	1.1573×10 <sup>-7</sup>	-	6.4102×10 <sup>-7</sup>	2.7653×10 <sup>-7</sup>	-
k <sub>1</sub>	-1.6506×10 <sup>-5</sup>	-7.0521×10 <sup>-6</sup>	-	-2.4210×10 <sup>-5</sup>	-1.2441×10 <sup>-5</sup>	-
l <sub>1</sub>	1.1581×10 <sup>-4</sup>	4.5560×10 <sup>-5</sup>	-	1.7493×10 <sup>-4</sup>	8.6503×10 <sup>-5</sup>	-
m <sub>1</sub>	-	-	1.2327×10 <sup>-3</sup>	-	-	1.2325×10 <sup>-3</sup>
n <sub>1</sub>	-	-	-2.2812×10 <sup>-2</sup>	-	-	-2.2812×10 <sup>-2</sup>
o <sub>1</sub>	-	-	9.2838×10 <sup>-1</sup>	-	-	9.2849×10 <sup>-1</sup>
p <sub>1</sub>	-	-	1.2659×10 <sup>-2</sup>	-	-	1.2646×10 <sup>-2</sup>
a <sub>2</sub>	1.2091×10 <sup>-3</sup>	1.1935×10 <sup>-3</sup>	-1.7359×10 <sup>-4</sup>	1.2728×10 <sup>-3</sup>	1.1774×10 <sup>-3</sup>	-1.7342×10 <sup>-4</sup>
b <sub>2</sub>	-1.3812×10 <sup>-2</sup>	-1.3427×10 <sup>-2</sup>	1.9158×10 <sup>-3</sup>	-1.4362×10 <sup>-2</sup>	-1.3280×10 <sup>-2</sup>	1.9142×10 <sup>-3</sup>
c <sub>2</sub>	3.7281×10 <sup>-2</sup>	3.5625×10 <sup>-2</sup>	-5.0058×10 <sup>-3</sup>	3.8176×10 <sup>-2</sup>	3.5319×10 <sup>-2</sup>	-5.0030×10 <sup>-3</sup>
d <sub>2</sub>	-5.0100×10 <sup>-2</sup>	-4.8322×10 <sup>-2</sup>	6.5870×10 <sup>-3</sup>	-5.2270×10 <sup>-2</sup>	-4.8017×10 <sup>-2</sup>	6.5820×10 <sup>-3</sup>
e <sub>2</sub>	5.6659×10 <sup>-1</sup>	5.4029×10 <sup>-1</sup>	-7.2532×10 <sup>-2</sup>	5.8645×10 <sup>-1</sup>	5.3837×10 <sup>-1</sup>	-7.2489×10 <sup>-2</sup>
f <sub>2</sub>	-1.5124	-1.4248	1.8909×10 <sup>-1</sup>	-1.55	-1.4234	1.8903×10 <sup>-1</sup>
g <sub>2</sub>	4.7018×10 <sup>-1</sup>	4.4028×10 <sup>-1</sup>	-5.7079×10 <sup>-2</sup>	4.8092×10 <sup>-1</sup>	4.3902×10 <sup>-1</sup>	-5.7124×10 <sup>-2</sup>

$h_2$	-5.2694	-4.8626	$6.4555 \times 10^{-1}$	-5.3428	-4.8357	$6.4612 \times 10^{-1}$
$i_2$	$1.4978 \times 10^{+1}$	$1.3667 \times 10^{+1}$	-1.7284	$1.4985 \times 10^{+1}$	$1.3559 \times 10^{+1}$	-1.7302
$j_2$	$-4.6210 \times 10^{-5}$	$-6.0782 \times 10^{-5}$	$5.7342 \times 10^{-4}$	$-5.8241 \times 10^{-5}$	$-6.4140 \times 10^{-5}$	$5.7318 \times 10^{-4}$
$k_2$	$5.2724 \times 10^{-4}$	$6.8855 \times 10^{-4}$	$-6.4228 \times 10^{-3}$	$6.5384 \times 10^{-4}$	$7.2382 \times 10^{-4}$	$-6.4210 \times 10^{-3}$
$l_2$	$-1.4247 \times 10^{-3}$	$-1.8419 \times 10^{-3}$	$1.7082 \times 10^{-2}$	$-1.7250 \times 10^{-3}$	$-1.9277 \times 10^{-3}$	$1.7081 \times 10^{-2}$
$m_2$	$1.8945 \times 10^{-3}$	$2.4521 \times 10^{-3}$	$-2.2418 \times 10^{-2}$	$2.3632 \times 10^{-3}$	$2.6004 \times 10^{-3}$	$-2.2410 \times 10^{-2}$
$n_2$	$-2.1760 \times 10^{-2}$	$-2.8061 \times 10^{-2}$	$2.4867 \times 10^{-1}$	$-2.6751 \times 10^{-2}$	$-2.9562 \times 10^{-2}$	$2.4860 \times 10^{-1}$
$o_2$	$5.9285 \times 10^{-2}$	$7.5892 \times 10^{-2}$	$-6.5500 \times 10^{-1}$	$7.1193 \times 10^{-2}$	$7.9359 \times 10^{-2}$	$-6.5493 \times 10^{-1}$
$p_2$	$-1.7399 \times 10^{-2}$	$-2.2287 \times 10^{-2}$	$2.1056 \times 10^{-1}$	$-2.1417 \times 10^{-2}$	$-2.3761 \times 10^{-2}$	$2.1062 \times 10^{-1}$
$q_2$	$2.0726 \times 10^{-1}$	$2.6388 \times 10^{-1}$	-2.4433	$2.5054 \times 10^{-1}$	$2.7787 \times 10^{-1}$	-2.4441
$r_2$	$-5.8755 \times 10^{-1}$	$-7.3793 \times 10^{-1}$	7.7184	$-6.8923 \times 10^{-1}$	$-7.6749 \times 10^{-1}$	7.7215

**Table 2. Constant coefficients of equations (1 to 4) for C3-I and C3-II column**

Constant	C3-I			C3-II		
	Creep	Elastic	Shrinkage	Creep	Elastic	Shrinkage
$a_1$	$2.6718 \times 10^{-9}$	$3.0752 \times 10^{-9}$	-	$2.4345 \times 10^{-9}$	$3.0413 \times 10^{-9}$	-
$b_1$	$-8.6274 \times 10^{-8}$	$-1.4797 \times 10^{-7}$	-	$-6.9826 \times 10^{-8}$	$-1.4379 \times 10^{-7}$	-
$c_1$	$-6.2324 \times 10^{-7}$	$1.3332 \times 10^{-6}$	-	$-9.2059 \times 10^{-7}$	$1.2182 \times 10^{-6}$	-
$d_1$	$-1.8834 \times 10^{-7}$	$-9.5500 \times 10^{-8}$	-	$-1.9806 \times 10^{-7}$	$-1.0314 \times 10^{-7}$	-
$e_1$	$1.0916 \times 10^{-5}$	$6.1732 \times 10^{-6}$	-	$1.1384 \times 10^{-5}$	$6.6033 \times 10^{-6}$	-
$f_1$	$-1.5099 \times 10^{-4}$	$-1.0925 \times 10^{-4}$	-	$-1.5576 \times 10^{-4}$	$-1.1496 \times 10^{-4}$	-
$g_1$	$-7.6820 \times 10^{-7}$	$-5.7125 \times 10^{-7}$	-	$-7.5635 \times 10^{-7}$	$-5.9215 \times 10^{-7}$	-
$h_1$	$1.7906 \times 10^{-5}$	$2.3986 \times 10^{-5}$	-	$1.5330 \times 10^{-5}$	$2.3654 \times 10^{-5}$	-
$i_1$	$1.0999 \times 10^{-3}$	$4.2525 \times 10^{-4}$	-	$1.1783 \times 10^{-3}$	$4.6390 \times 10^{-4}$	-
$j_1$	$9.4184 \times 10^{-7}$	$4.7852 \times 10^{-7}$	-	$9.7406 \times 10^{-7}$	$5.1253 \times 10^{-7}$	-
$k_1$	$-3.2752 \times 10^{-5}$	$-1.8388 \times 10^{-5}$	-	$-3.3346 \times 10^{-5}$	$-1.9075 \times 10^{-5}$	-
$l_1$	$2.4866 \times 10^{-4}$	$1.3416 \times 10^{-4}$	-	$2.5853 \times 10^{-4}$	$1.4166 \times 10^{-4}$	-
$m_1$	-	-	$1.2327 \times 10^{-3}$	-	-	$1.2327 \times 10^{-3}$
$n_1$	-	-	$-2.2812 \times 10^{-2}$	-	-	$-2.2812 \times 10^{-2}$
$o_1$	-	-	$9.2836 \times 10^{-1}$	-	-	$9.2842 \times 10^{-1}$
$p_1$	-	-	$1.2674 \times 10^{-2}$	-	-	$1.2653 \times 10^{-2}$
$a_2$	$1.2552 \times 10^{-3}$	$1.1642 \times 10^{-3}$	$-1.7334 \times 10^{-4}$	$1.2431 \times 10^{-3}$	$1.2694 \times 10^{-3}$	$-1.7353 \times 10^{-4}$
$b_2$	$-1.4181 \times 10^{-2}$	$-1.3150 \times 10^{-2}$	$1.9137 \times 10^{-3}$	$-1.4037 \times 10^{-2}$	$-1.4418 \times 10^{-2}$	$1.9162 \times 10^{-3}$
$c_2$	$3.7733 \times 10^{-2}$	$3.5022 \times 10^{-2}$	$-5.0023 \times 10^{-3}$	$3.7330 \times 10^{-2}$	$3.8744 \times 10^{-2}$	$-5.0095 \times 10^{-3}$
$d_2$	$-5.1767 \times 10^{-2}$	$-4.7768 \times 10^{-2}$	$6.5788 \times 10^{-3}$	$-5.1277 \times 10^{-2}$	$-5.1446 \times 10^{-2}$	$6.5848 \times 10^{-3}$
$e_2$	$5.8226 \times 10^{-1}$	$5.3716 \times 10^{-1}$	$-7.2467 \times 10^{-2}$	$5.7614 \times 10^{-1}$	$5.8108 \times 10^{-1}$	$-7.2553 \times 10^{-2}$
$f_2$	-1.5428	-1.4243	$1.8900 \times 10^{-1}$	-1.5249	-1.5524	$1.8925 \times 10^{-1}$
$g_2$	$4.7514 \times 10^{-1}$	$4.3632 \times 10^{-1}$	$-5.7108 \times 10^{-2}$	$4.7086 \times 10^{-1}$	$5.4275 \times 10^{-1}$	$-5.7128 \times 10^{-2}$
$h_2$	-5.2919	-4.8195	$6.4601 \times 10^{-1}$	-5.2271	-6.0792	$6.4642 \times 10^{-1}$
$i_2$	$1.4888 \times 10^{-1}$	$1.3551 \times 10^{-1}$	-1.7301	$1.4672 \times 10^{-1}$	$1.7187 \times 10^{-1}$	-1.7314
$j_2$	$-6.0347 \times 10^{-5}$	$-6.6702 \times 10^{-5}$	$5.7318 \times 10^{-4}$	$-6.0364 \times 10^{-5}$	$-6.6483 \times 10^{-5}$	$5.7331 \times 10^{-4}$
$k_2$	$6.7727 \times 10^{-4}$	$7.5051 \times 10^{-4}$	$-6.4217 \times 10^{-3}$	$6.7768 \times 10^{-4}$	$7.4790 \times 10^{-4}$	$-6.4241 \times 10^{-3}$
$l_2$	$-1.7864 \times 10^{-3}$	$-1.9919 \times 10^{-3}$	$1.7084 \times 10^{-2}$	$-1.7885 \times 10^{-3}$	$-1.9845 \times 10^{-3}$	$1.7091 \times 10^{-2}$
$m_2$	$2.4651 \times 10^{-3}$	$2.7188 \times 10^{-3}$	$-2.2410 \times 10^{-2}$	$2.4668 \times 10^{-3}$	$2.7109 \times 10^{-3}$	$-2.2412 \times 10^{-2}$

$n_2$	$-2.7879 \times 10^{-2}$	$-3.0756 \times 10^{-2}$	$2.4863 \times 10^{-1}$	$-2.7916 \times 10^{-2}$	$-3.0668 \times 10^{-2}$	$2.4870 \times 10^{-1}$
$o_2$	$7.4137 \times 10^{-2}$	$8.2099 \times 10^{-2}$	$-6.5511 \times 10^{-1}$	$7.4303 \times 10^{-2}$	$8.1874 \times 10^{-2}$	$-6.5533 \times 10^{-1}$
$p_2$	$-2.2602 \times 10^{-2}$	$-2.5003 \times 10^{-2}$	$2.1064 \times 10^{-1}$	$-2.2629 \times 10^{-2}$	$-2.4972 \times 10^{-2}$	$2.1061 \times 10^{-1}$
$q_2$	$2.6359 \times 10^{-1}$	$2.8956 \times 10^{-1}$	-2.4446	$2.6428 \times 10^{-1}$	$2.8952 \times 10^{-1}$	-2.4446
$r_2$	$-7.2304 \times 10^{-1}$	$-7.9168 \times 10^{-1}$	7.7230	$-7.2625 \times 10^{-1}$	$-7.9279 \times 10^{-1}$	7.7238

### References

- [1] M.J. Afshari, A. Kheyroddin, M. Gholhaki, Simplified sequential construction analysis of buildings with the new proposed method, *Structural Engineering and Mechanics*, 63(1) (2017) 77-88.
- [2] M.J. Afshari, A. Kheyroddin, M. Gholhaki, Simplified Time-Dependent Column Shortening Analysis in Special Reinforced Concrete Moment Frames, *Periodica Polytechnica Civil Engineering*, 62(1) (2018) 232-249.
- [3] M.J. Afshari, A. Kheyroddin, M. Gholhaki, The effect of constant and seasonal changes of ambient conditions on long-term behavior of high-rise concrete structures, *The Structural Design of Tall and Special Buildings*, 27(18) (2018) e1548.

Please cite this article using:

M. Jalilzadeh Afshari, A. Kheyroddin, A New Simplified Method of Time-dependent Column Shortening Analysis in Concrete Moment Frames, *Amirkabir J. Civil Eng.*, 50(5) (2018) 887-906.

DOI: 10.22060/ceej.2017.12625.5233

