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Fracture Modes of an Annular Crack in a Transversely Isotropic Solid

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ABSTRACT: The existence and extension of the cracks in structural materials is one of the issues to be considered to prevent the devastating effects of cracks. Cracks can be subjected to different types of fracture modes that considering these modes help us to predict the behavior of cracks. This paper investigates the effects of the fracture modes (opening, shearing and tearing) on the annular crack in an infinite transversely isotropic solid. In each mode, by substituting the boundary conditions into governing equations of the medium, the problem reduced to triple integral equations. With the aid of Hankel and Abel integral transforms, the triple integral equations reduced to two Fredholms integral equations which are amenable to numerical solutions. The inner and outer stress intensity factors of the annular crack are obtained for different ratios of inner-outer radius of the annular crack. Some limiting cases such as the penny-shaped crack and external crack are considered. From the results, it can be concluded that the stress intensity factors (SIFs) are independent of material properties; additionally, loads play major role in the variation of SIFs which may lead to change in the direction of crack extension.

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1- Introduction

It is common that all existing structural materials contain defects such as cracks which lead to a decrease in strength and lifetime of structures. Therefore, study of the interaction between an annular crack under different modes of loading and the surrounding medium should be considered.

Kassir and sih [1] obtained the exact SIF of penny-shaped crack and external crack under different loading conditions. The problem of internal loading of a flat annular crack located in an isotropic elastic solid was studied by Selvadurai and Singh [2]. Danyluk and Singh [3] considered the axisymmetric stress distribution in an infinite elastic solid containing an annular crack under axial torsion.

The objective of this paper is to consider an annular crack under different types of fracture modes in an infinite transversely isotropic solid medium. With the aid of Hankel and Abel integral transforms, the boundary conditions reduced to Fredholm integral equations which are amenable numerical treatments. Numerical results are demonstrated for different ratios of the annular crack length. Some limiting cases such as penny-shaped crack and external crack are covered.

2- Mode I

Consider an open annular crack of inner radius a and outer radius b, which is paralleled with the isotropic plane in an infinite transversely isotropic medium. The surfaces of the crack are subjected to a compressive stress. Due to the symmetry of the problem about the plane z=0 , it suffices to consider z≥0 . The mixed boundary conditions are as follows:

$$u_z(r,0)=0; \qquad 0\le r\le a \tag{1}$$

$$\sigma_{zz}(r,0)=-p(r); \qquad a<r<b \tag{2}$$

$$u_z(r,0)=0; \qquad b\le r<\infty \tag{3}$$

$$\sigma_{rz}(r,0)=0; \qquad 0<r<\infty \tag{4}$$

By inserting the related displacement and stress fields from [4], the conditions (1)-(4) can be expressed as the following triple integral equations:

$$\int_0^\infty A'(\xi)J_0(\xi r)\mathrm{d}\xi=0; \qquad 0\le r\le a \tag{5}$$

$$\int_0^\infty \xi A'(\xi)J_0(\xi r)\,\mathrm{d}\xi=\begin{cases} f_1(r) & 0<r<a \\ f(r)=-p(r)/C_1; & a<r<b \\ f_3(r) & b<r<\infty \end{cases} \tag{6}$$

$$\int_0^\infty A'(\xi)J_0(\xi r)\,\mathrm{d}\xi=0; \qquad b\le r<\infty \tag{7}$$

where $A'(\xi)$, $f_i(r)(i=1,3)$ are unknown functions and C_1 is a known coefficient. By using the procedure mentioned in [5] and avoiding details of calculations, the conditions (5)-(7) are reduced to the following Fredholm integral equations:

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$$F_1 \; s \; = -F \; s \; -\frac{2}{\pi}\int_b^\infty \frac{uF_3 \; u \; du}{u^2-s^2}; \qquad 0<r<a \tag{8}$$

$$F_3(s) = -F^*(s) - \frac{2s}{\pi}\int_0^a \frac{F_1(u)du}{s^2-u^2}; \qquad b<r<\infty \tag{9}$$

where

$$F_1(\mathrm{s}) = \int_s^a \frac{\lambda f_1(\lambda)\mathrm{d}\lambda}{\sqrt{\lambda^2-s^2}}; \qquad 0<s<a \tag{10}$$

$$F(\mathrm{s}) = \int_a^b \frac{\lambda f(\lambda)\mathrm{d}\lambda}{\sqrt{\lambda^2-s^2}}; \qquad a<s<b \tag{11}$$

$$F_2^*(\mathrm{s}) = \int_a^b \frac{\lambda f(\lambda)\mathrm{d}\lambda}{\sqrt{s^2-\lambda^2}}; \qquad a<s<b \tag{12}$$

$$F_3(\mathrm{s}) = \int_b^s \frac{\lambda f_3(\lambda)\mathrm{d}\lambda}{\sqrt{s^2-\lambda^2}}; \qquad b<s<\infty \tag{13}$$

The opening SIF can be defined as follows:

$$K_{\mathrm{I}}^a = \lim_{r\to a^-}\sqrt{2(a-r)}\,\sigma_{zz}(r,0) \tag{14}$$

$$K_{\mathrm{I}}^b = \lim_{r\to b^+}\sqrt{2(r-b)}\,\sigma_{zz}(r,0) \tag{15}$$

Finally, it can be shown that:

$$K_{\mathrm{I}}^a = \frac{2C_1}{\pi\sqrt{a}}F_1(a); \qquad K_{\mathrm{I}}^b = \frac{2C_1}{\pi\sqrt{b}}F_3(b) \tag{16}$$

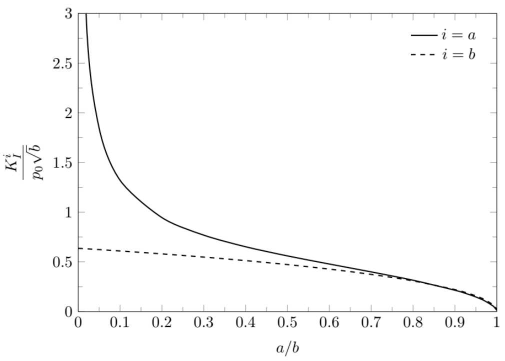

Figure 1. Normalized opening SIF under uniform loading

2- 1- Results

By considering the Equations (8), (9) and (16), it can be shown that the SIFs are independent from the properties of materials. The results of SIFs for inner and outer boundary of the annular crack versus the length of the crack under uniform pressure stress are shown in Figure 1. As it is clear, once $K_a^{\mathrm{I}} > K_b^{\mathrm{I}}$, the crack tends to extend toward the inner boundary.

3- Mode ɪɪ

In this case, the crack surfaces are loaded by equal and opposite shears p(r). The boundary conditions are:

$$u_r(r,0) = 0; \qquad 0\le r\le a, \quad b\le r<\infty \tag{17}$$

$$\sigma_{rz}(r,0) = -p(r); \qquad a<r<b \tag{18}$$

$$\sigma_{zz}(r,0) = 0; \qquad 0<r<\infty \tag{19}$$

Avoiding details of calculations, the equations (17)-(19) lead to the following Fredholm integral equations:

$$\begin{aligned} F_1(\mathrm{s}) &= -F(\mathrm{s}) + \frac{2}{\pi}\int_b^\infty \frac{s^2}{u(s^2-u^2)}F_3(\mathrm{u})\mathrm{d}u \\ &- \frac{2}{\pi}\int_b^\infty \frac{s}{2u^2}\log_e\left|\frac{s+u}{s-u}\right|F_3(\mathrm{u})\mathrm{d}u; \quad 0<s<a \end{aligned} \tag{20}$$

$$\begin{aligned} F_3(s) &= -F^*(s) - \frac{2}{\pi}\int_0^a \frac{s}{(s^2-u^2)}F_1(u)\mathrm{d}u \\ &- \frac{2}{\pi}\int_0^a \frac{1}{2u}\log_e\left|\frac{s+u}{s-u}\right|F_1(u)\mathrm{d}u; \quad b<s<\infty \end{aligned} \tag{21}$$

The shearing SIFs can be determined as follows:

$$K_{\mathrm{II}}^a = \frac{2C_2}{\pi a^{3/2}}F_1(a); \qquad K_{\mathrm{II}}^b = \frac{2C_2}{\pi b^{3/2}}F_3(b) \tag{22}$$

3- 1- Results

Similar to mode I, the shearing SIF is independent of material properties. The results of SIFs for inner and outer boundary of the annular crack versus the length of the crack under linear radial shear stress are shown in Figure 2. Obviously, as , the crack tends to extend toward the outer boundary.

4- Mode ɪɪɪ

In this case, the crack surfaces are subjected to equal and opposite twisting action. The boundary conditions are:

$$u_\theta(r,0) = 0; \qquad 0\le r\le a; \quad b\le r<\infty \tag{23}$$

$$\sigma_{\theta z}(r,0) = -p(r); \qquad a<r<b \tag{23}$$

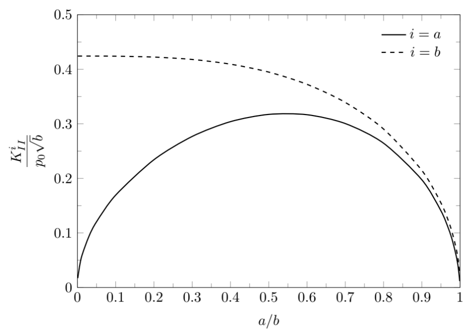

Figure 2. Normalized shearing SIF under linear loading

The governing equations and the results of SIFs are similar to the mode II.

5- Conclusions

An analytical treatment is presented for different loadings of an annular crack in an infinite transversely isotropic medium. By employing Hankel and Abel transforms and treatment of triple integrals, the mixed boundary value problem is reduced to the Fredholm integral equations. It is observed that in the three modes of fracture, the SIF is independent of material properties.

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