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Damage Detection of Structures using Transfer Function and its Singular Values

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ABSTRACT: In this paper, the application of Singular Variable Decomposition (SVD)-based principal component analysis (PCA) performed on truncated form of transfer function is demonstrated. Damage scenarios with light severity and distributed locations could be detected, localized and quantified using a one-step model updating. In many cases, it enhances the capability of FRF-based model updating with the presence of high noise levels and much less updating data. A numerical simulation on a truss has been validated to show the ability of this technique for damage detection.

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1- Introduction

The objective of this study is to demonstrate the application of SVD-based principal component analysis performed on moving windows of transfer function. It uses the sensitivities of measured responses in frequency domain, its singular values and right eigenvectors for Finite Element (FE) model updating in an efficient way, by developing a quasi-linear sensitivity equation of structural response. The benefit of applying Principal Component Analysis (PCA) for dynamical systems comes from its ability to detect and rank the dominant coherent spatial and frequency-dependent information of dynamic response.

The challenge of using modal parameters of structure for damage detection is that they may change highly by variation of operational conditions and structural uncertainties. Hence, most vibration-based model updating techniques only give good results in well-controlled laboratory conditions preventing the noise to mask the information on the damage state of structure. In this regard, proper selection of measured frequency points for updating of noisy data with low damage levels were have been addressed in this paper. Finally, validation of this method is evaluated using numerical simulations and a beam experiment.

2- Model Updating Procedure

Since no exact solution exists here, this over-determined system of equations should be solved by Least Square numerical methods such as “Brut force” inversion, Pseudo-inversion using SVD, Gaussian elimination solution or QR factorization. The quality of predicted damage depends on several factors including the sensor types and locations, excitation types and locations, measurement and modeling error, updating frequency points, appropriate weighting techniques to avoid forming ill-conditioned systems, observability of unknown parameters and numerical methods used for solution of the system of equations. In this paper, some of individual equations in were omitted because of low sensitivities to the unknown parameters and magnification of adverse effects of measurement errors. For avoiding the least-squares solution to be dominated by the equations with the largest coefficients, both sides of equations were multiplied by a scale factor as a weighting approach. Therefore, in this paper, each row of sensitivity matrix equation was be weighted by the inverse of its second norm.

3- Updating Frequency Range

Given a set of sensor locations and frequency points, for parameter estimations, it is necessary to have the highest value of change in the response due to changes in the unknown structural parameters. These points are located around resonance frequencies where amplitude of vibration is larger and changes rapidly by damage. At frequency ranges of low vibration amplitude, structural response may highly be affected by noise levels proving incapability of these points for a robust model updating. On the other hand, using excitation frequency very close to resonant frequency makes the model very sensitive to damping factors and its measurement errors.

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4- Incomplete Measurement

Considering modal and spatial truncation in measuring natural frequencies and mode shapes of damaged structure caused by practical limitations, $H_d(\omega)$ is approximated using the analytical mode shapes of the intact structure φ, to alleviate this source of error in sensitivity equation. Defining the measured natural frequency of the damaged structure as Ω_{di} and measured damping loss factor as ξ_{di}, approximated transfer function has been introduced such that Equation 1:

$$H_d(\omega) \cong \sum_{i=1}^{nm} \frac{\phi_i\phi_i^T}{\Omega_{ndi}^2-\omega^2+2j\xi_{di}\Omega_{di}\omega} + \sum_{i=nm+1}^{m} \frac{\phi_i\phi_i^T}{\Omega_{ni}^2-\omega^2+2j\xi_i\Omega_i\omega} \qquad (1)$$

Where nm is the number of the measured natural frequencies. This approximation is realistic because it is possible to measure natural frequencies with high accuracy. The second term has been used to improve the formulation needed for considering incomplete measurement effects by increasing the convergence rate.

Ω_n and Ω_{nd} are natural frequencies of intact and damaged structure respectively. Model updating frequency ranges are starting frequency points around considered mode shapes. They may change during further updating iterations with a fixed size to make model converge to a realistic result. Although very high mode shapes are very sensitive to light damage severities, they can't be measured in real structural applications. But incompleteness of real data can be handled by the concept used in Equation 1. On the other hand, natural frequencies corresponding to high modes of analytical structure can be determined solving the numerical eigenvalue problem. Then, frequency ranges around the resulted high modes were applied to the completed real data for model updating.

5- Numerical Simulation

The presented damage detection algorithm was applied to a benchmark truss structure modeled numerically using finite element method with axial element. Esfandiari et al. used the same truss for FRF-based model updating [1]. Truss geometry, element numbers, and excitation and measurement nodes are showed in Figure 1. Eliminating constraint DOFs on the support nodes, DOF numbers assigned to each node, follow the relation (2*(node number -1)-1) for the horizontal direction and (2*(node number -1)) for the vertical direction. The unknown parameters are axial stiffness of elements, EA where A is the cross-section area of truss element and E is the Young's modules. Elements are made from steel martial with the Young's module of 200Gpa, mass density of 7850 kg/m^3 .

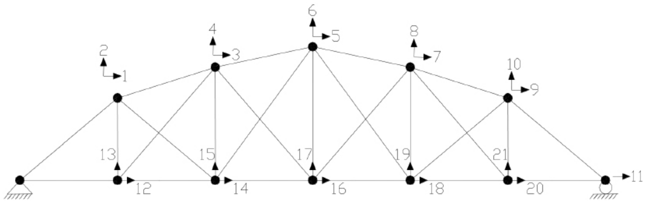

Figure 1: Numerical Truss

6- Discussion

Three snapshots of FRF matrices were used in SVD-based updating process while in FRF-based updating, number of snapshots of FRF matrices were equal to number of updating frequency points. So, much less updating data were used in SVD-based method which leaded to more accurate converging. It should be mentioned that although each snapshot of FRF matrix in SVD-based method contained more column data in comparison with its corresponding FRF matrix, numerical simulation can be faster in SVD-based method because for each updating iteration, all the calculations should be done for each snapshot of matrices.

The objective of this study is to demonstrate the application of SVD-based principal component analysis performed on moving windows of transfer function. It uses the sensitivities of measured responses in frequency domain, its singular values and right eigenvectors for FE model updating in an efficient way, by developing a quasi-linear sensitivity equation of structural response. The benefit of applying PCA for dynamical systems comes from its ability to detect and rank the dominant coherent spatial and frequency-dependent information of dynamic response.

The challenge of using modal parameters of structure for damage detection is that they may change highly by variation of operational conditions and structural uncertainties. Hence, most vibration-based model updating techniques only give good results in well-controlled laboratory conditions preventing the noise to mask the information on the damage state of structure. In this regard, proper selection of measured frequency points for updating of noisy data with low damage levels were have been addressed in this paper. Finally, validation of this method is evaluated using numerical simulations.

7- Conclusions

In this paper, a model updating method using SVD-based principal component analysis (PCA) is presented. It was concluded that PCA of frequency domain data based on the SVD is a useful tool in linear structural dynamics, in general, because it results in vectors and matrices which are similar to modal properties of systems. It decomposes dataset to left singular vectors, right singular vectors and singular values indicating spatial, temporal and energy content of data respectively.

Damage detection and localization methodology utilized two separate moving windows with fixed lengths for constructing two separate sensitivity matrices. It result in large differences between objective functions of the damaged structure and intact structure to increases the chance of successful prediction of the damage location and severity. In this regard, FE model of a numerical truss and a tested beam were validated using this method. To avoid a non-linear and non-monotones objective function, the transfer function of the damaged structures should be approximated using measured natural frequencies of damaged structure and analytical mode shapes of intact structure. Incompleteness of modal data plays an important role for detecting large damages, while measurement errors have the main influence when there is small damage to identify. The approximated transfer function has an acceptable evaluation of FRF especially near natural frequencies, where updating frequencies were chosen in these regions either.

Using SVD-based updating, it has been demonstrated that main modal features of the structure including natural frequencies and FRFs match well with the experimental model considering distributed damage locations. Handling high noise levels and more accurate results than FRF-based updating are major points of this algorithm. Also, this method doesn't need to predict the potential damage locations at first to narrow the search space for better damage quantification in the next step. Future work could extend the model updating approaches to incorporate more dependencies between sensor's data interpretation in frequency domain.

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