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Analytical Approach for Boundary Layer Control around Elliptical Cylinders by Suction

A.R. Kabiri-Samani[1*], M. H. Golmohammadi[2]

1- Associate Prof., Civil Eng. Dept., Isfahan University of Technology
2- PhD Candidate, Civil Eng. Dept., Isfahan University of Technology

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ABSTRACT

Control of boundary layer is a common problem in river engineering to reduce the scouring around bridge piers. There are different approaches for controlling the boundary layer such as: motion of the solid wall, accelerating flow by blowing and suction, elimination of flow separation using streamlined profiles and continuous suction. There is not any information about the solution of boundary layer equations coupled with continuous suction around elliptical sections. The present work is to study the laminar boundary layer developed by 2-D incompressible flow around circular and elliptical sections. The boundary layer equations and its control using continuous suction are surveyed analytically. Compariosion between the present results and those of other investigations indicates that the present analytical model is a good tool to predict the velocity profiles around an elliptical cylinder. The velocity and shear stress distribution, the boundary layer thickness, drag force and the friction coefficient distribution are determined and presented with and without control. The control procedure presented here could be applied for different speeds of flow suction.

KEYWORDS:

Boundary layer control, Suction, Analytical modeling, Laminar, Elliptical section.

[*] Corresponding Author, Email: akabiri@cc.iut.ac.ir

1- BRIEF INTRODUCTION

Flow behavior around a stationary cylinder and control of boundary layer are common problems in viscous fluid flows. Prandtl (1904) was the first who studied the effects of fluid viscosity and found that the effects are just limited to a thin layer close to an object, so-called “boundary layer” [1]. Brodestsky (1923) presented an analytical model to study the flow behavior in a flow with high Reynolds number around a stationary cylinder [2]. Forenberg (1980), Behr et al. (1994), Lee et al. (2004) have presented different methods to study the effects of fluid velocity on drag force and pressure coefficient around a stationary cylinder located in a visous fluid flow [3-5]. One of the most common methods to control the boundary layer in the viscous fluid flows around a stationary cylinder such as; bridge piers is the flow suction [1]. Jain (1960), applied the Pohlhausen’s considerations and presented new correlations to study the flow behavior around a stationary cylinder under the condition of flow suction [6-7]. Regarding to the former studies, an analytical approach is developed for controlling the boundary layer around an elliptical cylinder by means of flow suction herein.

2- METHODOLGY

Considering the Navier Stockes equations as well as the elliptic curves in a normal coordinate system, the governing differential equation is:

$$\nu \frac{d^2u}{d\eta^2} - v_0 \frac{du}{d\eta} = UU' = f \qquad (1)$$

where u and v are the tangential (s) and normal (η) velocity components respectively and U is the potential velocity around the elliptical cylinder in normal coordinate system.

This equation is a 2nd order ordinary differential equation (ODE) which is resolved by the method of changing variables. Equation (2) gives the solution of this ODE, including the velocity distribution around an elliptical cylinder under the flow suction.

$$\frac{u(\eta)}{U(s)} = \frac{-v_0^2 + U'\delta v_0 - \nu U' e^{\frac{v_0}{\nu}\eta} + \nu U'}{v_0^2\left(e^{\frac{v_0}{\nu}\delta} - 1\right)} + \frac{v_0 - U'\delta}{v_0\left(e^{\frac{v_0}{\nu}\delta} - 1\right)} e^{\frac{v_0}{\nu}\eta} + \frac{U'}{v_0}\eta + \frac{\nu U'}{v_0^2} \qquad (2)$$

where v0 is the constant suction velocity along the normal direction (η), δ is the boundary layer thickness and ν is the kinematic viscosity.

Calculating the shear stress (τ_w) on the elliptical cylinder leads to the following equation:

$$\tau_w = \mu \frac{\partial u}{\partial \eta}\Big|_{\eta=0} = U(s)\rho\left(\frac{v_0 - U'\delta}{e^{\frac{v_0}{\nu}\delta} - 1} + \nu \frac{U'}{v_0}\right) \qquad (3)$$

3- Results

For verifying the results, obtained velocity distribution was simplified and the resulting velocity profiles around a flat plate and a circular cylinder under the flow suction were achieved. Accordingly, the results were compared to those of Schlichting [1]. Furthermore, results of the present study were compared with the results of empirical equations presented by Jain [6] and Pohlhausen [7]. Results show that the new analytical approach is a good tool to predict the flow hydraulic characteristics in controlling the boundary layer around the elliptical cylinders under the flow suction. The obtained velocity profile was also applied to a practical example. Figure (1) shows the velocity distribution around an elliptical cylinder of a = 0.1 and b = 0.2 m with U_0=0.8 m/s and a suction velocity of v_0=0.01U_0.

According to the present results, flow suction leads to reduce the boundary layer thickness and controls the flow separation. Figure (2) shows that while the the suction velocity increases, the boundary layer thickness decreses significantly.

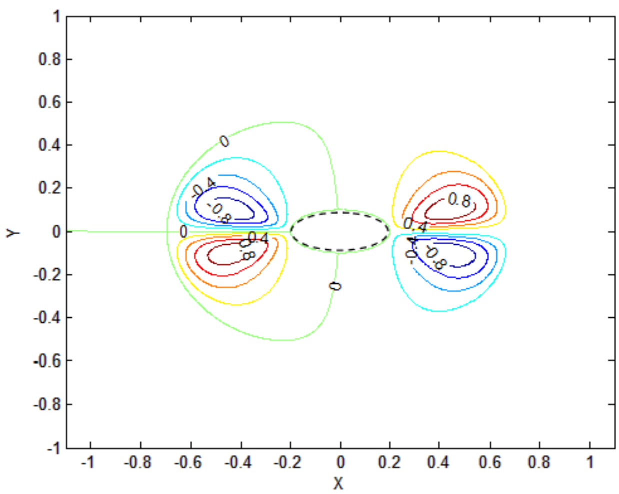

Figure 1. Velocity profile around an elliptical cylinder (flow is from right to left)

4- MAIN CONTRIBUTIONS

The major highlights and outcomes of the present study are:

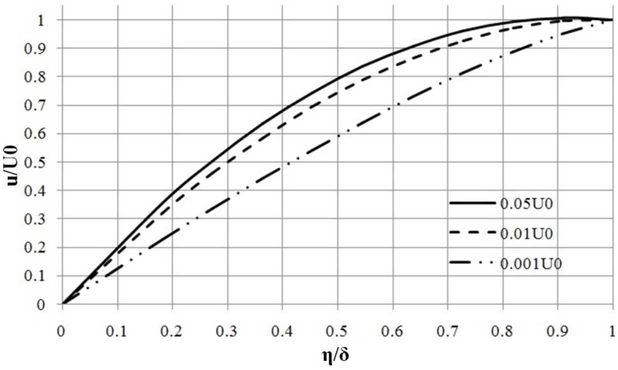

Figure 2. Velocity profiles for θ=90 on an elliptical cylinder for different suction velocities

- An analytical approach was developed to study the velocity distribution around an elliptical cylinder under the flow suction.
- The objective of the present study was control of boundary layer, which has not be considered around the elliptical cylinders previously.
- Present analytical correlation might be a good tool to predict the velocity distribution around the elliptical cylinders under the flow suction.
- Shear stress, drag force coefficient, displacement thickness and stream function were achived based on the obtained velocity profile.

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