



Analytical Modeling of Load-Deflection Curve in Reinforced Concrete Beams

S. Hajighasemali¹, A. Ramezaniapur^{2*}, M. Kashefzadeh³

1- Assistant Professor, Faculty of Engineering, Islamic Azad University, Roudehen branch, Tehran

2- Professor, Faculty of Civil Engineering, Amirkabir University of Technology, Tehran

3- M.Sc., Construction Management, UTM University of Malaysia, Skudai, Johor, Malaysia

(Received 14 October, 2013, Accepted 13 September, 2014)

ABSTRACT

Two effective parameters in determining the length of reinforcements in the reinforced soil slopes are, the one, the length of reinforcement located in the active zone till to the location of failure surface and the second, the length of reinforcement located after the failure surface. Generally, the first one is calculated based on the angle of failure wedge by Rankin method. In this method the effect of reinforcement on the location of failure surface is ignored, while the presence of reinforcement is effective. In order to assess the location of the failure surface, the horizontal slice method based on Spencer assumption is used. In this method, slippery mass with the presence of reinforces is divided into a number of horizontal slices parallel to reinforcement direction. Inter-slice forces are computed by using Spencer basic rules. Earthquake load is affected on the center of each slice by horizontal and vertical pseudo-static coefficients. In the presented method, unlike the other existing methods, all of the critical slip surfaces are examined and are reinforced. In this paper, Genetic Algorithm (GA) optimization method is used to optimize the objective function for the produced non-circular slip surface of each horizontal for the safety factor of one. By comparing the results of Genetic Algorithm optimization approach introduced in this research with the results of the other investigators for the same geometry, material properties and loadings of the slopes it is indicated that the introduced and utilized method is more critical for the estimation of the length of reinforcements and the design of reinforcements with the proposed method is more reliable.

KEYWORDS:

Reinforced Slope, Optimization, Genetic Algorithm, None Circular Failure Surface

* Corresponding Author, Email: aaramce@aut.ac.ir

1- Introduction

Although LVDTs produce precise results, they only provide one-dimensional information and are best suited to laboratory use. Moreover, its measurement range is limited and accuracy degrades significantly outside its linear range, which restricts the magnitude of measured deflection. Moreover, destructive tests damage LVDT, prohibiting accurate deflection measurement when the beam is close to failure. So, insufficient data are obtained from the load-deflection curve of a beam. Therefore, there is a need to provide the required data with an alternative solution. Many research have been conducted to determine the load-deflection relationship of the RC members [1, 2, 3, 4, 5, 6, 7, 8]. In spite of several research, literature suffers from lack of a research regarding deflection measurement in the case of damaged measurement devices.

2- Research Significance

Literature shows that beam members have an elasto-plastic behavior in the failure moment. So, the relationship between the load and deflection can be obtained by an analytical method (Hajjighasemali et. al., 2008). Therefore, the aim of the study is to propose an analytical model to find the relationship between the load and deflection of the RC beams when the measurement instruments are damaged through the test.

Displacement ductility index is calculated by Equation 1 and is considered as the basis of ductility comparison between analytical and experimental models.

$$\mu_{\Delta} = \frac{\Delta_u}{\Delta_y} \tag{1}$$

Where Δ_u and Δ_y are ultimate and tension steel yield deflection, respectively and are defined in load-deflection curves.

3- Experimental Modeling

An experimental model was initially conducted in a controlled indoor testing facility in the Department of Civil Engineering laboratory at the Roudehen Azad University of Iran. Three beam specimens were built in a laboratory. The average strength of concrete at 28 days was 26.8 MPa. Three 1100 mm long RC beams with 100×150 mm rectangular cross section and were provided with stirrups. For steel strains measurements, a strain gauge was mounted on the middle of each bar.

3- 1- Four point bending test

Detail of the flexural loading is given in Figure 1. Deflections at the first point and middle span of the beams were measured using LVDT. During the test, the total force, steel strains and deflections of two points of beams were recorded.

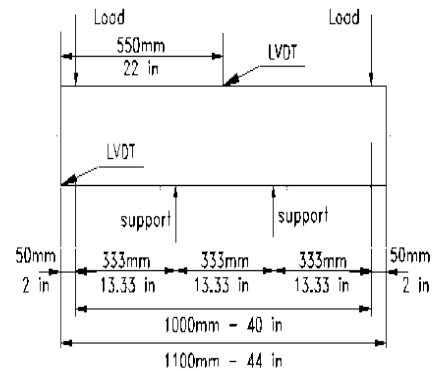


Figure 1. Four points bending test

4- Analytical Model

In general, concrete behaviour is divided into three stages: elastic stage, elastoplastic stage and plastic stage. In order to analyse the Beam’s behaviour, the beam is initially divided into two parts as shown in Figure 2.

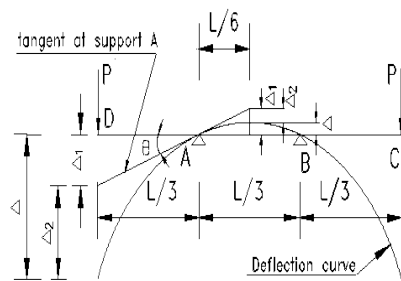


Figure 2. Deflection curve of the beam during the flexural loading

For the first point of beam, the deflection is obtained by equation 2:

$$\Delta = \Delta_1 + \Delta_2 \tag{2}$$

For the middle span point of the beam, the deflection is computed by equation 3.

$$\Delta = \Delta_1 - \Delta_2 \tag{3}$$

Where Δ is total deflection of beam under load, Δ_1 is the rigid deflection of beam defined by tangent to the deflection curve at the support A, and Δ_2 is tangential deviation in the direction perpendicular to the undeformed axis of the beam.

Within the elastic stage, the two Delta values

are considerable. In the second stage (elasto-plastic stage) the value of Δ_1 is far bigger than that of Δ_2 . In this stage Δ_1 represents the elastic behavior and Δ_2 indicates the elasto-plastic behavior. As the load is increased the value of Δ_1 is much greater than the Δ_2 and therefore the effect of Δ_2 is insignificant comparing with . It is due to large rotation of hinged support in the elastic- plastic stage.

Δ_1 in Equations 2 and 3 is obtained as follows:

For the first point of the beam Equation 1 is employed:

$$\Delta_1 = \theta \times \frac{L}{3} \tag{4}$$

And for the middle span point:

$$\Delta_1 = \theta \times \frac{L}{6} \tag{5}$$

Where θ is rotation of hinged support at A, and L is length of the beam.

Moreover, Δ_1 is obtained from multiplication of hinged support by length of the point from the hinged support.

The curvature is obtained by Equation 6:

$$\kappa = \frac{1}{\rho} = 2\theta \times \frac{L}{3} \tag{6}$$

Where κ is the Curvature and ρ is radius of curvature. From the Equation 6, the rotation of hinged support is computed as in Equation 7:

$$\theta = \frac{3\kappa}{2L} \tag{7}$$

The curvature is obtained from the reinforcement strains distribution in cross section recorded by strain gauge during the test, as given in Equation 8.

$$\kappa = \frac{\epsilon_s + \epsilon_{sc}}{d - e} \tag{8}$$

Where:

ϵ_{st} = tension reinforcement strain, ϵ_{sc} = compression reinforcement strain

d: distance from compression face to tension reinforcement, e: distance from compression face to compression reinforcement. Δ_2 is obtained from the second moment- area theorem as following:

For the first point of beam, Equation 9 is employed:

$$\Delta_2 = \frac{P L^3}{216 E_c I_c} \tag{9}$$

Where P is the Point load, E_c is Concrete modulus

of elasticity and I_c is effective moment of inertia.

In addition, according to ACI-318-0814, the value of E_c for normal weight of concrete is obtained from Equation (11) (MPa):

$$E_c = 4700 \sqrt{f'_c} \tag{10}$$

Where f'_c is specific compressive strength of concrete.

In order to calculate I_e , the ACI-318-0814 recommends the use of Branson's equation to account for the effective moment of inertia after cracking (Equation 12):

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g \tag{11}$$

Where; M_{cr} is cracking moment, M_a is Maximum moment in the beam, I_g is Gross transformed section moment of inertia and I_{cr} is cracked section moment of inertia

The cracking moment M_{cr} of the beam is obtained from the Equation 13:

$$M_{cr} = \frac{f_r I_g}{y_t} \tag{12}$$

Where; y_t is distance from neutral axis of gross section, neglecting reinforcement, to tension face, and f_r is the modulus of rupture of concrete which is defined as Equation 14:

$$f_r = 0.62 \sqrt{f'_c} \tag{13}$$

For the sake of simplicity, the relation between stress and strain is assumed to be linear as far as $f = 0.40$.

5- Results And Discussion

The load-deflection curves for the first points of beams 1, 2 and 3 are given in Figures 5, 6 and 7, respectively. Comparison of the results of experimental model and analytical model for the middle span points of the beams are shown in Figures 8, 9 and 10. Results show that the deflection of the first points of the beam is about 23 millimeters and results of analytical model vary between 20mm and 8mm. Moreover, deflection of the middle span points in the test is about 11mm while on the other hand; it varies within 8mm and 11mm for the analytical model. In addition, the calculated measures of toughness of the beams for the first point and middle span points are shown in Table 1. Difference between results of the

analytical and experimental model for beams 1, 2 and 3 are 2.15, 7.57 and 8.68 respectively. Comparison of ductility for the experimental and analytical models is indicated in Table 2. As it can be seen in the Table, the difference between the results of the analytical and experimental model for beams 1, 2 and 3 are 9.68, 7.40 and 9.05, respectively. These results imply that the analytical results have good convergence to experimental results with less than 9 percent error.

Table 1. Comparison of the Strength analyses of beams for experimental and analytical models

Difference of analytical and experimental models (%)	Difference of analytical and experimental models (%)	Beam
2.15	7.30	Beam 1
7.57	8.20	Beam 2
8.68	6.85	Beam 3

Table 2. Ductility of the Beams

Difference of analytical and experimental models (%)	μ_{Δ} Experimental model	μ_{Δ} Analytical model	Beam
9.68	6.20	5.60	Beam 1
7.40	4.32	4.00	Beam 2
9.05	4.09	3.72	Beam 3

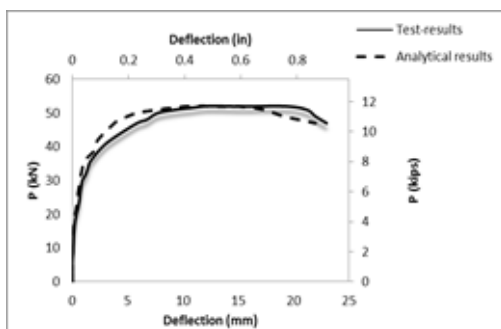


Figure 3. comparison of the analytical and experimental results for first point of beam 1

6- Conclusion

In this study, the relation between load and deflection of R/Ce beams was studied. Comparison of the results of the experimental and analytical models shows that the deviation of the analytical model is less than 9 percent and therefore this method has a good compatibility with the results of the experiment. Findings of the study confirm that if the instruments of

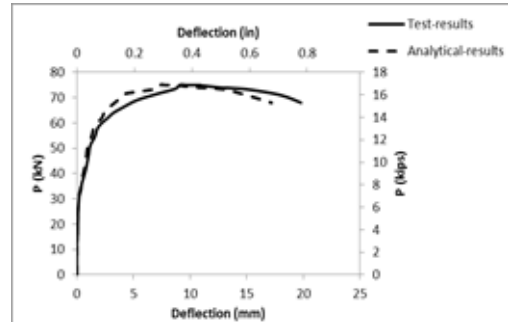


Figure 4. comparison of the analytical and experimental results for first point of beam 2

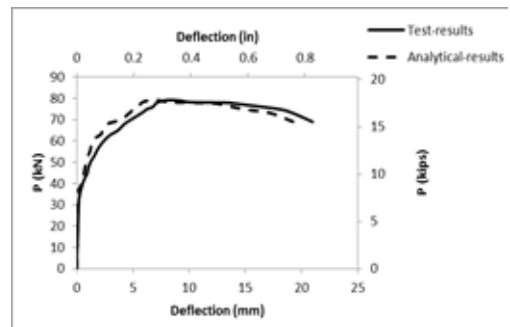


Figure 5. comparison of the analytical and experimental results for first point of beam 3

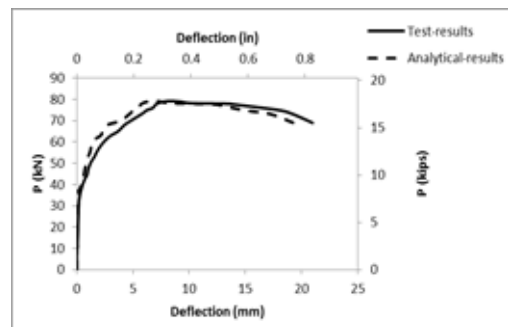


Figure 6. comparison of the analytical and experimental results for middle span point of beam 1

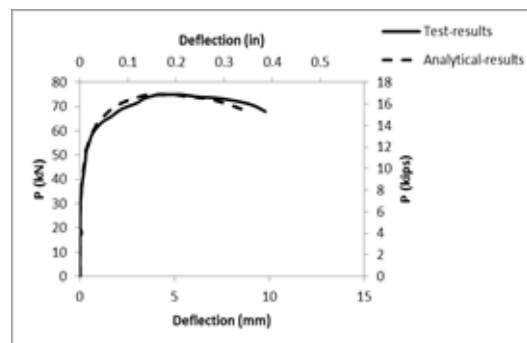


Figure 7. comparison of the analytical and experimental results for middle span point of beam 2

tests are damaged during the test, the load-deflection can be obtained using an analytical method alongside a gage, and the analytical model is enough accurate to find the load-deflection curve. However, ACI 318-

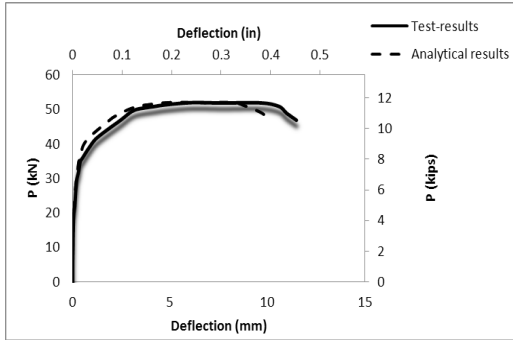


Figure 8. comparison of the analytical and experimental results for middle span point of beam 3

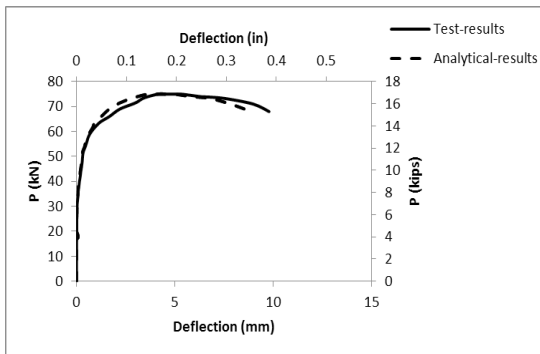


Figure 9. comparison of the analytical and experimental results for middle span point of beam 2

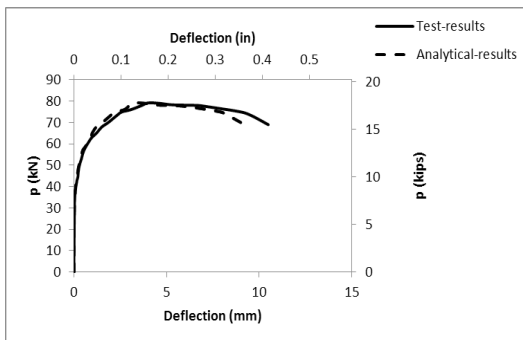


Figure 10. comparison of the analytical and experimental results for middle span point of beam 2

08 was the standard applied in this study in order to propose the analytical model and other standards can be employed for future studies.

7- REFERENCES

[1] Bayrak O., Sheikh A. Sh., “Plastic hinge analysis”, Journal of structural engineering, vol. 127, No. 9, pp. 1092-1100, September, 2001.
 [2] Abadalla H. A., “Evaluation of deflection in concrete members reinforced with fiber reinforced

polymer (FRP) bars”, composite structures, vol. 6, No. 1, pp. 63-71, April 2002.

[3] Cocchetti G., Maier G., “Elastic- plastic and limit state analysis of frames with softening plastic- hinge models by mathematical programming”, International Journal of solids and structures, vol. 40, No. 25, pp. 7219-7244, December, 2003.

[4] Bischoff P., “Reevaluation of deflection prediction for concrete beams reinforced with steel and fiber reinforced polymer bars” Journal of structural engineering, vol. 131, No. 5, pp. 752-768, 2005.

[5] Scott M. H., Fennes G. L., “Plastic hinge integration methods for force- based beam- column elements”, Journal of structural engineering, ASCE, vol. 132, No. 2, pp. 244-252, February 2006.

[6] Inel M., Ozman H. B., “Effect of plastic hinge properties in nonlinear analysis of reinforced concrete buildings”, Engineering structures, vol. 28, No.1, pp. 1494 – 1502, September, 2006.

[7] Sonam Jamtsho, “Geometric Modelling of 3D Range Cameras and their Application for Structural Deformation Measurement”, Student thesis, University of Calgary, 2010.

[8] Kalkan I., “Deflection prediction for reinforced concrete beams through different effective moment of inertia expressions”, International journal of research & development, vol. 2, No. 1, pp. 72-80, January, 2010.

[9] He R., Zhong H., “Large deflection elaso- plastic analysis of frames sing the weak- form quadrature element method”, Finite element in analysis and design, vol. 50, pp. 125-133, 2012.

[10] Hee Sun Kim, Yeong Soo Shin “Flexural behavior of reinforced concrete (RC) beams retrofitted with hybrid fiber reinforced polymers (FRPs) under sustaining loads”, Composite structures, vol. 93, No. 2, pp. 802-811, 2011. 10

[11] Hajighasemali S., Ramezaniapur A. A., Lotfi, vol., “Flexural strength and numerical simulation of ASR affected beams”, 13th international conference on Alkali aggregate reaction in concrete, Norway, 2008. 11

