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Computation of Discretization Error Using the Rule of Gradient Recovery and Adaptive Refinement of Elements

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ABSTRACT

Since the beginning of modeling physical events by computers, the finite element method has been firmly accepted as one of the most efficient general techniques the numerical solution of a variety of problems encountered in engineering. But no one has provided an answer to accurately determine the discretization error value in analyzing a structural problem using finite element method and there is almost no accessible tool to select suitable sizes for elements and proper types of solutions and the size of each element is selected based on experts' judgments.

The present paper is an attempt to present a closed-form solution for three-node triangular elements in order to estimate the discretization error in continuous domains by using the rule of gradient recovery and h-refinement adaptivity. Computing the discretization error and diagnosing the suitability of the elements size are possible by the closed-form solution presented.

KEYWORDS:

Adaptive Refinement; Rule of Gradient Recovery; Error norm; Adaptive Finite Element Method; Error Post-Processor; Discretization

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1- BRIEF INTRODUCTION

One of the most important weaknesses of finite element method is the lack of awareness of the suitability of elements size in the analysis. In finite element method, there is almost no tool for users to diagnose the suitability of elements size and the accuracy of solution. And the size of elements is often chosen based on experts' judgments, So that the experts' judgments sometimes make the problem too complex and large to solve by ordinary software packages. The lack of awareness of the discretization error value during the analysis process is another weakness of finite element method. In this method, also the location of the error is unknown. In recent years, remarkable advances have been made to overcome the weaknesses of finite element method. One advance is using error post-processors and adaptive refinement.

In the early 1980s, Demkowicz, Bank and Weiser's research on recovery methods resulted in a wide range of error estimators. In the late 1980s, Zhu and Zienkiewicz presented a simple error estimator and adaptive procedure for practical engineering analysis. In the early 1990s, post-processing of error was established and then was directed to general problems. Most methods presented in the above–mentioned years are considered as"recovery-based methods". In recovery methods, using the concept of error norm, the error value is evaluated by the comparison of the finite element approximation gradients and smoothed gradients in each element. In the late 1990s, Zhu and Zienkiewicz improved their method to the superconvergent patch recovery (SPR) [1],[2],[3]. In 1997, Boroomand and Zienkiewicz used stresses in Gauss points and equilibrium in patches instead of superconvergent points and the concept of the norm for the entire domain, respectively. [4],[5]

The aim of this paper is to present a closed-form solution for three-node triangular elements in order to estimate the discretization error in continuous domains by using the rule of gradient and adaptive refinement of the elements. The weaknesses of general finite element methods are overcome by the closed-form solution presented. Furthermore, a method is introduced to diagnose the suitability of the element's size and the error value in the discretization process by this closed-form solution.

2- The CONCEPT OF NORM AND ERROR ESTIMATION

According to the definition, the energy norm for the problems of elasticity with the above behavior equation is expressed by the following relationship

$$\|u\| = \left[\int_\Omega \sigma^T D^{-1} \sigma \, d\Omega\right]^{1/2} \tag{1}$$

Like the above relationships, the error norm of energy in a linear elastic problem is computed by the following relationship:

$$\|e\| = \left[\int_\Omega (\sigma - \sigma_h)^T D^{-1} (\sigma - \sigma_h) d\Omega\right]^{1/2} \tag{2}$$

And the relative error percentage of the energy norm can be computed by the following relationship:

$$\eta = \left[\frac{\|e\|^2}{\|u\|^2 + \|e\|^2}\right]^{1/2} \times 100\ \% \tag{3}$$

Error estimation by the relationship (3) is possible only if the exact solutions are available. But since the exact solution is not usually available in advance, it is necessary to estimate the exact solution to compute the error. An efficient and simple method to access to the exact solutions and to estimate the error value is the rule of gradient recovery. In some references, the rule of gradient recovery is known as project or variational recovery process. In the gradient recovery method, it is assumed that the nodal stresses are unknown and are represented by $\bar{\sigma}$. Having the nodal stresses, the internal stresses (σ^*) can be computed by appropriate shape function N_σ.

$$\sigma^* = N_\sigma . \bar{\sigma} \tag{4}$$

According the research carried out, if stress shape functions (N_σ) are selected as displacement shape functions (N), the stress distribution is more accurate. Therefore, the smoothed stress relationship within the elements is as follows:

$$\sigma^* = N . \bar{\sigma} \tag{5}$$

It is thus required that after the final analysis is completed the condition that

$$\eta < \bar{\eta} \tag{6}$$

Be satisfied for the whole domain, where $\bar{\eta}$ is the maximum permissible error. In practical conditions, $\bar{\eta}$ is considered less than 10 %. If $\eta > \bar{\eta}$, the error is more than the permissible error and mesh generation must be refined to obtain an acceptable response.

3- ADAPTIVE REFINEMENT BY USING LOCAL ERROR ESTIMATION

In general, local refinement operations of mesh generation are carried out based on one of the following methods:

(i) h-refinement

(ii) p-refinement

(iii) h-p-refinement

Each of the above-mentioned refinement methods can be used in one of the following forms:

- Refinement by using enrichment mesh generation;
- Refinement by using automatic mesh generation;

In enrichment mesh generation method, the primary discretization structure is maintained during the refinement process and the refinement operations are performed on elements whose relative errors are impermissible. But in automatic mesh generation method, the primary discretization structure is changed and a new mesh generation in harmony with the local relative error is designed.

In this research, h- refinement and enrichment mesh generation have been used to determine the ultimate load and possible failure path. Most research on h- refinement adaptivity method, have mostly carried out on rectangular elements (four nodes). But in this research, three-node triangular elements have been used due to the existence of closed-form solutions for them.

4- CONCLUSION

In this paper, first, the finite element method was presented as an efficient method for the solution of a variety of problems encountered in engineering. But the finite element method has weaknesses such as the lack of awareness of the suitability of elements size and the lack of information about the discretization error value in the analysis. Therefore, the adaptive finite element method was presented as a powerful technique in order to overcome the weaknesses of general finite element method.

The advantages of the adaptive finite element method over the general finite element method are:

(i) Computation of overall relative error (η) based on the rule of gradient recovery;

(ii) Computation of the local relative error (ξ_i) for all elements;

(iii) Adaptive refinement in the discretization process;

The investigation of several examples demonstrated that the acceptable solution obtained does not depend on the type of the norm selected. In other words, it is possible to access the acceptable solution by the selection of any type of norm. In this research, for the computation of error and access to a wise mesh among the energy norm, L2 norm for stress and L2 norm for strain, the energy norm was used and acceptable results were obtained.

In this paper, also, the h-refinement with the enrichment mesh generation was used for the refinement of mesh. The advantage of h-refinement with the enrichment mesh generation over other methods is to easily access to the final response, such that only one element can be used to start the solution process and after repeating the solution for several times by the software packages prepared, a wise mesh and an acceptable response can be obtained.

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