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A High Resolution Finite Volume Scheme with a Voronoi Mesh for Dam Break Simulation

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ABSTRACT

A high resolution finite volume method for solving the shallow water equations with voronoi mesh is developed via applying MATLAB software in this paper. The scheme is formally uniformly second order accurate and satisfies the maximum principles. The model is verified by comparing the model output with condition of anti-symmetric and circular dam break with the documented results. For more investigation, we utilized SPSS statistical software. Very good agreement has been achieved in the verification phase. It can be considered as / to be an efficient implement for the computation of shallow water problems, especially concerning those having discontinuities. A simple example of the collapse of water supply reservoir in a valley is used to demonstrate the capability of the model. The presented model is able to resolve the shocks, handling, complex geometry, including the influence of steep bed slopes.

KEYWORDS

Finite Volume Method, Voronoi Mesh, Dam Break, High Resolution Local Lax–Friedrich Scheme.

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1- INTRODUCTION

The design, construction and operation of dams, they may be broken under unpredictable events such as huge floods, war or earthquake [1]. The shallow water equations (SWEs) are conventionally used to describe the unsteady open channel flow such as dam break [2]. These equations are named as Saint Venant equations for one-dimensional (1D) problem and also include the continuity and momentum equations for two dimensional (2D) studies [3], [4]. Erpicum et al. presented a 2D finite volume (FV) multiblock flow solver, which was able to deal with the natural topography variation [5]. Baghlani utilized a combination of the robust and effective flux-difference splitting (FDS) and flux-vector splitting (FVS) methods to simulate the dam break problems based on FVM on a Cartesian grid. Zhang and Wu developed a hydrodynamic and sediment transport model for the dam break flows. The 2D SWEs were solved based on the FVM with an unstructured quadtree mesh grid. Singh et al. developed a 2D numerical model to solve the SWEs for the simulation of dam break problems. Chang et al. proposed a meshless numerical model to investigate the shallow water (SW) dam break in 1D open channel. Shakibaeinia and Jin developed a new mesh-free particle model based on the weakly compressible MPS (WC-MPS) formulation for modeling the dam break problem over a mobile bed. Sarveram and Shamsai investigated the dam break problem in converge and diverge rectangular channels in the unsteady stance using Saint Venant equations and a quasi-Lagrangian method. This paper attempts to present a novel development for 2D dam break problems. A high-resolution FVM is employed to solve the SWEs on unstructured Voronoi mesh. The local Lax-Friedrichs (LLxF) scheme is used for the estimation of fluxes at cells and the numerical approximation of hyperbolic conservation laws.

2- METHODOLOGY

In this article, the studied domain was discretized using unstructured Voronoi meshes with MATLAB software. The continuity and momentum equations of the SW can be written in different forms depending upon the requirements of the numerical solution of governing equations. The 2D SWEs with source terms are given in the vector form considering a rigid bed channel as follows:

$$U_t + F_x + G_y = S \tag{1}$$

$$U_t = \begin{bmatrix} h \\ hu \\ hv \end{bmatrix} = \begin{bmatrix} h \\ q_x \\ q_y \end{bmatrix} \tag{2}$$

$$F_x = \begin{bmatrix} q_x \\ \frac{{q_x}^2}{h} + \frac{1}{2} g h^2 \\ \frac{q_x q_y}{h} \end{bmatrix} \tag{3}$$

$$G_y = \begin{bmatrix} q_y \\ \frac{q_x q_y}{h} \\ \frac{{q_y}^2}{h} + \frac{1}{2} g h^2 \end{bmatrix} \tag{4}$$

Although safety criteria are considered through

$$S = \begin{bmatrix} 0 \\ gh(S_{0x} - S_{fx}) \\ gh(S_{0y} - S_{fy}) \end{bmatrix} \tag{5}$$

Where: U is the vector of conserved variables; F and G are the flux vector functions; S is the vector of source terms; u and v are velocity components in x and y directions, respectively. h is also the water depth and (S_{0x},S_{0y}), are bed slopes. Discretization of the equation 1 is performed applying FVM with unstructured voronoi mesh in domain $\boldsymbol{\Omega}$ (Figure 1).

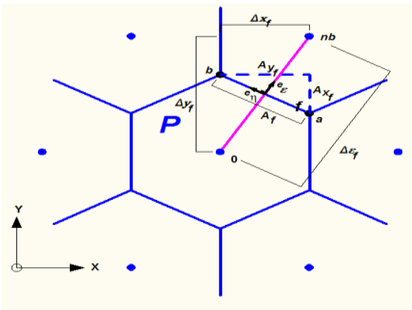

FIG. 1. The 2D SCHEMATIC VORONOI MESH CELL

$$\frac{\partial}{\partial t}\int_v u\, d_v + \int_v \vec{\nabla}.H(u)d\vartheta = \int_v S(u)d_v \tag{6}$$

Where H(u) is the input and output flux to voronoi cell that contains G (u) and F (u) functions in x and y directions. Discrete equation can be written as Equation (7)

$$U_p^{n+1} = U_p^n - \frac{\Delta t}{A}\sum_f [n_1.\ F(u)_\varepsilon.A_f + n_2.\ G(u)_\varepsilon.A_f] + S(u)\,\Delta t$$

Where: $F(u)_\varepsilon$, , $G(u)_\varepsilon$ are voronoi cell normal flux vectors and Δt is the interval that is computed from courant number clause.

The Local Lax-Friedrichs high order scheme in voronoi mesh is as follows:

(8)

$$U_p^{n+1} = U_p^n - \frac{\Delta t}{A} + \sum(G_f(v)_{nbout}.A_f\,.n_{2f}) - \sum(G_f(v)_{nboin}.A_f\,.n_{2f})] \quad + S(u)\,\Delta t$$

Validation

1. Two dimensional anti-symmetric dam break test

Many researcher, have implemented this test. The channel that is located in horizontal bed has 200 meters length, 200 meters width.

3- CONCLUSION

In the present research, a novel and friendly user code was written. It is observed that the LLxF scheme along with the FVM method by using unstructured Voronoi grid is a suitable combination in order to simulate 2D dam break problems.

There was not, significant numerical dispersion problem or nonphysical alternation. This consequence was predicted because of combination of FVM and voronoi mesh. The computed P-VALUES demonstrates high ability of the FVDBC code. The results of FVDBC code with voronoi mesh presented higher efficiency and precision than rectangular meshes.

Thus it is better to use the voronoi mesh in numerical discrete equations because not only able to complicated geometries but also can give accurate results in lower computational effort.

It contains an anti-symmetric cut with 75 meters width. The channel and dam domain are assumed frictionless. The upstream water depth is proposed 10 meters and the downstream water depth is assumed 5 meters. In this test we proposed 40*40 node

points and the suggested total time for comparing values is 7.2 second. The resuls are shown in table1.

TABLE 1. P-VALUE RESULTED FROM T TEST

Reference	Wang & Lu		Yuling & Wenli	
Mesh type	voronoi	Rectangular	voronoi	Rectangular
P-VALUE	0.973	0.967	0.959	0.948

2- Two dimensional dam break circular test

In this test we have considered a frictionless and horizontal rectangular domain that has 200 meters length and 200 meters width. The initial conditions consisted of two states separated by a circular discontinuity.

The computational grid consists of 40 *40 cells and the radius of the circle r = 50 meters and it is centered at x = 100 meters.

The water depth outside the circle is 1 meter deep and inside the circle is 10 meters deep. The results are shown in table2.

TABLE 2. P-VALUE RESULTED FROM T TEST

Reference	Wang&Lu	
Mesh type	Voronoi	Rectangular
P-VALUE	0.996	0.974

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