Improving the geometric regularity of the free-form space structure with random points (Ati Center Mashhad Space structure)

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ABSTRACT

Geometric regularity in free-form space structures plays an important role in structural performance, constructability, and economic efficiency by reducing member-length diversity and increasing the repetition of similar lengths. The structure comprises 176 nodes, 477 members, and 14 member-length categories. The research aims to improve geometric regularity by relocating nodes on the free-form surface and reducing the number of distinct member lengths. First, a continuous surface was reconstructed from the original structural nodes. Then, 176 points were randomly distributed over the surface, and the initial network was generated using Delaunay triangulation. To improve the nodal arrangement, the circular traversal algorithm and octopus optimization were evaluated using three approaches: minimizing member-length deviations, a proposed objective function based on reducing members with non-repeated lengths, and a combined objective function. The results showed that the proposed objective function reduced the number of member-length categories from 14 to 10 and decreased the proportion of members with distinct lengths from 40% to 25%. Furthermore, several angular and area-based indicators improved, and the structural weight decreased by 26.2%. However, improvements in quantitative indicators did not necessarily produce the most visually regular arrangement. This study These findings highlight the need to develop a comprehensive indicator for evaluating geometric regularity in space structures by simultaneously considering geometric characteristics, visual perception, and structural performance.

KEYWORDS

Free-form Space Structure, Geometric Regularity Improvement, Circular Traversal Algorithm, Octopus Optimization, Delaunay Triangulation.

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1. Introduction

Space frame structures have gained considerable attention in modern construction due to their ability to cover large spans, relatively low weight, and flexibility in architectural design [1]. Advances in computational design tools have made it possible to create complex and free-form geometries. However, the geometric complexity of these structures may lead to greater diversity in member lengths, difficulties in connection design, and increased fabrication complexity. Therefore, achieving an appropriate level of geometric regularity while maintaining architectural quality and structural performance remains a major challenge in their design. Geometric regularity in space frame structures is evaluated based on the degree of similarity among member lengths, angles, and panel areas. Various indices have been introduced to assess this property; however, a unified and comprehensive criterion for defining and measuring geometric regularity has not yet been established [3]. Early indices in this field include the length ratio and shape ratio, which were proposed to evaluate the geometric similarity of polygonal elements [5-4]. Other criteria, such as length deviation, variance, range, and standard deviation, have also been employed to investigate the dispersion of geometric properties [6]. Previous studies have developed various approaches to improving the geometric regularity of structures, including genetic algorithms [7], methods for evaluating and comparing regularity indices [8], metaheuristic algorithms [9], particle swarm optimization (PSO) [10], and evolutionary techniques [11]. Furthermore, Delaunay triangulation is considered an effective method for generating triangular meshes with favorable geometric quality and can be employed in the development of structural grids [12].

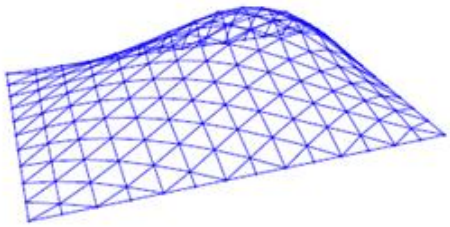

Figure 1. Free-form space frame structure of Ati Center in Mashhad, modeled in Rhino by the engineers of Ofogh Noor Space Structures Company (Model M1).

In this research, 176 randomly distributed points were generated on the parametric surface to maintain the original node count and enable direct comparison. The resulting Delaunay mesh was optimized to improve geometric regularity and reduce member-length diversity and structural weight. Modeling and optimization were performed in Rhino 7 using Grasshopper, Kangaroo], and Octopus (version 0.6.3.0).

2. Methodology

In the first stage, the continuous surface of the Ati Center structure was parametrically reconstructed in Rhino using the Grasshopper plugin to enable controlled displacement of nodes across the surface. While maintaining the original number of structural nodes, 176 points were generated, including 172 randomly distributed points and four fixed corner points. The initial structural network was then generated using Delaunay triangulation. Subsequently, the proposed Circular Traversal algorithm was employed to improve the nodal arrangement through controlled point displacement over the surface. In addition, the Octopus multi-objective optimization plugin in Grasshopper was used to optimize the objective functions. Two main approaches were investigated: first, an objective function based on the deviation of member lengths, aimed at reducing length dispersion and increasing network uniformity; and second, the proposed objective function (Equation (1)), which, through eliminating repeated lengths and identifying distinct member lengths, reduces both length dispersion and the number of distinct member-length types. The Create Set component in Grasshopper was used to extract distinct lengths. A combined approach was also examined to simultaneously evaluate different geometric regularity criteria.

$$\frac{\sqrt{\frac{\sum \left(L_i - L_{av}\right)^2}{n}}}{L_{av}} \tag{1}$$

After generating the optimized networks, geometric regularity indices were calculated in three categories: length-based, angular, and area-based. Finally, the initial model and the selected optimized model were modeled, analyzed, and designed in SAP2000 to investigate the effects of geometric modifications on internal forces, required cross-sections, and structural weight. To ensure a valid comparison, the overall dimensions, boundary conditions, support conditions, and loading were kept identical across all models, and the nodes located along the structural edges were modeled as fixed supports. Dead, snow, and wind loads of 70, 150, and 86 decanewtons per square meter, respectively, were applied. The structural members were designed using 140 × 60 mm rectangular hollow steel sections in accordance with the AISC 360-19 standard.

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generated from randomly distributed points (Figure 1), and two optimized models, M1 (Figure 1) and M2 (Figure 2), was evaluated using Length, Shape, and areal indices. Model M1 was optimized using the first objective function, while Model M2 was optimized using the second objective function.

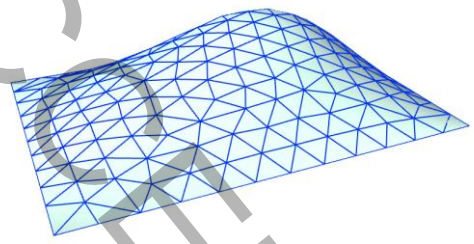

Figure 2. Model M2

The results showed that the structure generated from random points exhibited the greatest geometric irregularity, whereas both optimized models improved geometric order. Model M2 performed better in most longitudinal indices, reducing the number of distinct member-length types from 14 to 10, while Model M1 achieved better results in some areal indices. The original structure also showed better performance in certain angular indices. Structural analysis further indicated that Model M2 reduced the total structural weight by approximately 26.2% compared with the original structure. Overall, the findings demonstrate that improving geometric order can enhance constructability and reduce structural weight. However, comprehensive evaluation of geometric order requires combining geometric indices with structural performance criteria.

3. Results and Discussion

The geometric regularity of the structures was assessed using three indices—longitudinal, angular, and areal and compared across four configurations, including the original case-study structure (Figure 1), the structure generated from randomly distributed points (Figure 2), and two optimized models, M1 (Figure 16) and M2 (Figure 3). Model M1 was obtained through optimization using the first objective function, whereas Model M2 was obtained using the second objective function.The results showed that the structure generated from random points exhibited the greatest geometric irregularity. In contrast, both optimized models improved the geometric order indices. Model M2 demonstrated better performance in most longitudinal indices, including a reduction in the range of member lengths and a decrease in the number of distinct member-length types from 14 to 10. Meanwhile, Model M1 achieved slightly better results in some areal indices. The original structure also maintained better performance in certain angular indices. Structural analysis further indicated that Model M2, despite a slight increase in the maximum axial force, enabled the use of lighter cross-sections and reduced the total structural weight by approximately 26.2% compared with the original structure. The comparison of the three order indices indicates that geometric regularity is a multidimensional concept, as each index evaluates a different aspect of geometric regularity. Consequently, no single configuration consistently outperforms the others across all indices. Although Model M2 performs better in most longitudinal indices and structural weight reduction, Model M1 shows slightly better performance in some areal indices, while the original structure retains advantages in certain angular indices. These findings highlight the importance of considering multiple geometric and structural criteria when evaluating the order of free-form space-frame structures. Furthermore, developing a comprehensive composite order index that integrates geometric characteristics, structural performance, and constructability could provide a more reliable basis for evaluating and comparing optimized configurations in future research.

4. Conclusions

This research proposed an optimization approach based on the circular traversal algorithm and the Octopus optimization algorithm to improve the geometric order of free-form space-frame structures. The results demonstrated that optimizing node arrangements can reduce member-length diversity, improve structural constructability, and decrease material consumption and overall structural weight. Model M2 achieved the best performance in most longitudinal indices, reducing the number of distinct member-length types from 14 to 10 and the total structural weight by approximately 26.2% compared with the original structure. However, the differences among the longitudinal, angular, and areal indices indicate that geometric order is a multidimensional concept that cannot be adequately evaluated using a single criterion. Therefore, developing a comprehensive and standardized composite order index that incorporates geometric characteristics, structural performance, and constructability **is** recommended for future research.

5. References

[1] Tsuboi, Y. (1984). Analysis, design and realization of space frames. Bulletin of the International Association of Shell and Spatial Structures, 15, 84–85.

[2] Office of Deputy for Strategic Supervision, Bureau of Technical Execution System. (2010). Code of Practice for Skeletal Steel Space Structures. Islamic Republic of Iran Vice Presidency for Strategic Planning and Supervision, No. 400. (in Persian).

[3] Behnejad, S. A. (2018). Geometrical Data for Lattice Spatial Structures: Regularity, Historical Background

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and Education. PhD thesis, Spatial Structures Research Centre, Department of Civil and Environmental Engineering, University of Surrey, 1–217.

[4] Haghnazar, R., Nooshin, H., & Golabchi, M. (2014). Improving the regularity of geodesic domes using the concept of stepping projection. International Journal of Space Structures, 29(2), 81–95.

[5] Nooshin, H., Ishikawa, I., Disney, P. L., & Butterworth, J. W. (1997). The Traviation Process. Journal of the International Association for Shell and Spatial Structures, 38(125), 165–176.

[6] Kouroiwa, Y. Regulariazation of Structural Forms Using Genetic Algorithms. PhD thesis, Space Structures Research Centre, Department of Civil Engineering, University of Surrey. Published by ProQuest LLC (2019), 1–350.

[7] Nooshin, H., Kuriowa, Y., & Disney, P. L. (1999). A genetic method for regularization of structural configurations. IASS Congress, 20–24 September, Madrid, Spain, 1–10.

[8] Piórkowski, A., Mazurek, P., & Gronkowska-Serafin, J. (2015). Comparison of assessment regularity methods dedicated to isotropic cells structures analysis. Advances in Intelligent Systems and Computing, 1–11.

[9] Massah, S. A., & Ahmadi, H. (2017). Regularizing structural configurations by using meta-heuristic algorithms. Geomechanics and Engineering, 12(2), 197–210.

[10] Mohades, A., Goodarzi, M., & Forghani-Elahabad, M. (2020). Particle swarm optimization method in optimization of grid shell structures. Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, 7(1), 1–6.

[11] Goodarzi, M., Mohades, A., & Forghani-Elahabad, M. (2021). Improving the gridshells' regularity by using evolutionary techniques. Mathematics, 9, 440.

[12] Elshakhs, Y. S., Deliparaschos, K. M., Charalambous, T., Oliva, G., & Zolotas, A. (2024). A comprehensive survey on Delaunay triangulation: Applications, algorithms, and implementations over CPUs, GPUs, and FPGAs. IEEE Access, 12, 12562–12585.

[13] Zhang, Y., Liu, Y., Lee, T. U., Rezaee Javan, A., Pietroni, N., & Xie, Y. M. (2023). Reducing the number of different nodes in space frame structures through clustering and optimization. Engineering Structures, 284, 116016.

[14] Hemmatian Dehkordi, P., Mohades Khorasani, A., Goodarzi, M., & Chaichi Raghimi, M. (2023). Improving the regularity of the Bidboland Khuzestan space dome using circular traversal algorithm and genetic optimization. Journal of Structural and Construction Engineering, 55(4), 873–892.

[15] Favilli, A., Laccone, F., Cignoni, P., Malomo, L., & Giorgi, D. (2024). Geometric deep learning for statics-aware grid shells. Computers & Structures, 292, 107238.

[16] Bi, M., Liu, Y., Xu, T., He, Y., Ma, J., Zhuang, Z., & Xie, Y. M. (2024). Clustering and optimization of nodes, beams and panels for cost-effective fabrication of free-form surfaces. Engineering Structures, 307, 117912.

[17] Zhong, Y., Liu, Y., Lu, H., Feng, R., & Xie, Y. M. (2025). Design and optimisation of grid shells for maximising both structural performance and panel similarity. Engineering Structures, 329, 119820.

[18] Duan, Y., Yang, Y., Zhang, T., Wei, Y., Shi, C., Zhang, S., Li, P., Jiao, Z., & Gao, W. (2026). A two-stage multi-objective optimization framework for sunspace design: Separating baseline building and sunspace effects on energy, comfort, and daylight in cold-region rural dwellings. Journal of Building Engineering, 117, 114678.

[19] R. Vierlinger, "Octopus," *Grasshopper3D*. [Online]. Available: https://www.grasshopper3d.com/group/octopus. [Accessed: Nov. 26, 2026].

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