Hybrid Boundary-Finite Element Method for Modeling Artificial Freezing in Soil Environment Including Rock Type Inhomogeneities

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ABSTRACT

This research proposes a hybrid numerical method, for modeling the development of artificial ground freezing in soil containing subsurface inhomogeneities. In this approach, by combining the Boundary Element Method (BEM) and employing time-independent fundamental solutions, appropriate boundary integral equations are developed. The volume integrals arising from the presence of dynamic terms are incorporated into the equations using the Finite Element Method (FEM). For this purpose, a type of boundary-finite element, combining a quadratic boundary element with a three-node triangular finite element, was developed, and the solvable forms of the final equations were presented. Subsequently, by implementing the hybrid method into a computational algorithm, its accuracy and efficiency was evaluated and validated by solving several benchmark examples. Finally, in a parametric study, the application of the hybrid method for modeling the development of artificial freezing in saturated soil containing an inhomogeneities in the form of an unsaturated rock mass is described. The effects of varying the cross-sectional area of the inhomogeneities and its distance from the freeze pipe were evaluated. The results of the parametric study indicate that the presence of the inhomogeneities reduces the volume of the freeze bulb by up to 20%. Furthermore, inhomogeneities with a circular cross-section were more limited the development of freezing compared to a square shape.

KEYWORDS

Artificial ground freezing, numerical modeling, hybrid method, Circular and square inhomogeneities, Boundary-Finite Element Method

1. Introduction

Generally, freezing in soil and rock environments can be classified into two main domains: natural freezing and artificial freezing [1]. Natural freezing occurs due to climatic changes and the formation of successive freeze-thaw cycles, particularly during the cold season. This process mainly involves the surface layers of the soil and, except in polar regions, rarely penetrates more than 10 meters below the ground surface. However, secondary effects of freezing, such as contraction-expansion cycles and soil volume changes of up to 9% are among the factors that highlight the importance of this phenomenon in various geographical regions, especially during cold seasons [2]. On the other hand, artificial freezing is human-made, achieved using temperature-lowering equipment in the soil. The primary objective of this type of freezing is the temporary improvement and strengthening of weak and low-bearing capacity sections of the soil, such that by freezing and creating stronger bonds between soil particles, the soil mass can exhibit greater resistance to applied loads [1, 2]. Zhou et al. (2014) described the use of the finite element method in modeling heat diffusion equations and developed a three-phase model consisting of soil, water, and ice lenses [3]. Alzoubi et al. (2020) presented a review study introducing various modeling methods for artificial freezing propagation. According to their findings, studies on freezing phenomena modeling in soils are very limited, and existing models are not capable of accounting for all the complex conditions governing freezing processes [4]. One important issue in soil freezing propagation is the formation of ice lenses. This phenomenon alters the thermodynamic properties of soil. Ren et al. (2023) used a machine learning-based model to assess and predict the formation of ice lenses in soil and compared their results with available laboratory and field data [5].

A review of the technical literature on modeling methods for freezing propagation in soil reveals that the heat diffusion equation is one of the most fundamental governing equations for modeling this phenomenon. In this research, a hybrid method is proposed to solve this equation. In this process, by simultaneously employing the boundary element method to close distant boundaries and the finite element method for meshing regions related to artificial freezing the accuracy and speed of numerical modeling are improved. Subsequently, using these equations, a heterogeneous soil environment under artificial freezing is modeled, and the effects of cross-sectional geometry and spacing of subsurface inhomogeneities on the propagation of freezing in the soil are assessed. The results of this research demonstrate that the hybrid method is an accurate approach for modeling freezing in semi-infinite soil environments and can be optimally utilized for modeling problems with complex geometries and conditions.

2. Methodology

The heat diffusion equation in an infinite rod was first developed by Fourier [1, 2]. By extending this equation to a two- dimensional environment, following equation is can be presented [1]:

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\sigma\rho\frac{\partial T}{\partial t} \tag{1}$$

Where k, σ, and ρ represent the thermal conductivity, specific heat capacity, and density of the medium, respectively. Moreover, in this equation, T denotes the temperature variation per unit area as a function of time t. defining the boundary-volume element using the combined three-node triangle element and quadradic boundary element along with the use of time-independent fundamental solutions, Equation (1) is ultimately transformed into the following solvable form:

$$\begin{aligned} & c^i T^i+\sum_{j=1}^{NE}\int_{\Gamma_j} q^*.\boldsymbol{f}.d\Gamma_j\left[T\right]-\sum_{j=1}^{NE}\int_{\Gamma_j} T^*.\boldsymbol{f}.d\Gamma_j\left[q\right] \\ & =\frac{\sigma\rho}{k}\sum_{j=1}^{NI}\int_{\Omega_j} T^*.\boldsymbol{f}.d\Omega_j\left[\dot{T}\right] \end{aligned} \tag{2}$$

where T^* and q^* are the time-independent fundamental solutions related to temperature and flux respectively [6] and T and q are the temperature and flux in domain, Ω bounded by Γ. c^i is the constant value that can be calculated based on the smaller angle between two adjacent boundary elements. Also, in Eq.2, NE represents the number of boundary elements and NI denotes the number of internal triangle elements. In all integrals $\boldsymbol{f}$ is the shape function. The compact matrix form of the Eq.2 can be represented as follows:

$$\left[H\right]\left\{T\right\}+\left[G\right]\left\{q\right\}=\left[I\right]\left\{\dot{T}\right\} \tag{3}$$

In above equation, $[H]$, $[G]$ and $[I]$ are the coefficient matrixes. Base on figure (1), Eq.3 can be applied to the soil model with internal inhomogeneities. In this regard the domain is divided to two distinct models: first model includes the soil environment with a cavity instead of inhomogeneity and second model is the domain of inhomogeneity itself. By applying Eq.3 for each model and combining the resultant equations considering continuity of temperature and flux at interface boundaries, the final solvable form of the equations can be presented as follows:

$$\begin{bmatrix} H_{11} & H_{12} & 0 \\ 0 & H_{21} & H_{22} \end{bmatrix} \begin{Bmatrix} T_{11} \\ T_{12} \\ T_{22} \end{Bmatrix} + \begin{bmatrix} G_{11} & G_{12} & 0 \\ 0 & -G_{21} & G_{22} \end{bmatrix} \begin{Bmatrix} q_{11} \\ q_{12} \\ q_{22} \end{Bmatrix} = \begin{bmatrix} I_{11} & 0 \\ 0 & I_{22} \end{bmatrix} \begin{Bmatrix} \dot{T}_{11} \\ \dot{T}_{22} \end{Bmatrix} \qquad (4)$$

where, all element with index 11 are related to first model and all elements with index 22 are for second (inhomogeneity) model. In Eq.4 index 12 and 21 are the values that are related to interface boundaries.

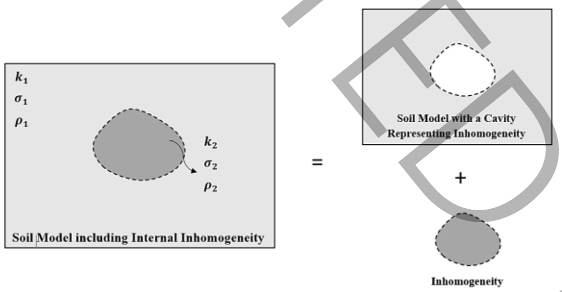

Figure 1. Soil model including inhomogeneity divided to two distinct models

3. Model

Given the application of the hybrid model in analyzing various types of soil environments with inhomogeneities this section focuses on a soil medium containing a subsurface inhomogeneity in the form of an unsaturated rock mass. The soil medium is subjected to artificial ground freezing using a freezing pipe under thermal shock down to 0°C. According to Figure (5), the geometric specifications and governing boundary conditions of this environment are presented. As shown in the figure, the ground surface boundary is at a normal temperature of 23°C due to heat exchange with the environment. The boundaries of the freezing pipe are also modeled as a constant temperature of 0°C. It should be noted that at the start of the computation, all nodes and points of the problem are in equilibrium at a normal temperature of 23°C. After the analysis begins, the temperature of the freezing pipe is reduced at its boundaries using an exponential function of the form $T(t) = A(e^{-1/st} - 1)$, where A is the reduction amplitude equal to 23°C, and s is the reduction rate equivalent to 2000 units per second.

For this model it is assumed that soil is a type of compacted sand and inhomogeneity is an unsaturated rock. Table (1) presents the thermodynamic properties of the unsaturated rock and sand soil used in this study.

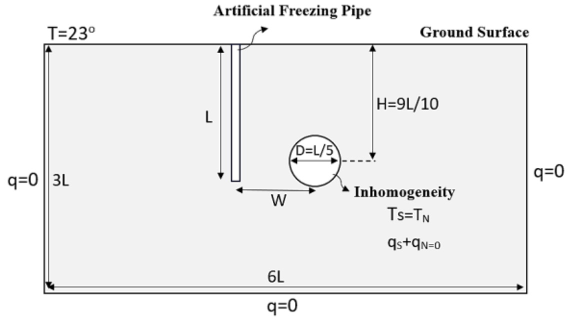

Figure 2. Geometric specifications and boundary conditions of the artificial ground freezing model, including a subsurface inhomogeneity and a freezing pipe of length L.

Table 1. The geometric specifications of the soil environment and inhomogeneity [7]

Model	Dr(%)	Temp(^{0}C)	ρ	k	σ
Soil	80	23	2148	2.5	1350
Rock	-	23	2568	0.034	1532

4. Results and Discussion

Figure (3) presents temperature variations over time along the centerline of the freezing pipe. In this figure, temperature variations as a function of distance from the freezing pipe are shown for evaluation points at distances of 0.1 to 9 meters from the pipe (denoted as R=0 to R=9) for the case where subsurface inhomogeneity is not considered. Alongside, results for the case where a square-shaped inhomogeneity is embedded in the soil layer are displayed using symbols RR=0 to RR=9 for evaluation points at 0.1 to 9 meters from the freezing pipe. Figures (3-a) to (3-c) illustrate the effect of varying the inhomogeneity distance on the temperature. As clearly observed in these figures, the square-shaped inhomogeneity significantly influences the temperature propagation around the freezing pipe. For instance, in Figure (3-a), when the inhomogeneity is at its closest distance to the freezing pipe (W=0.12L), the temperature near the pipe (RR=0) decreases at a slower rate in the initial steps compared to the case without inhomogeneity, with final freezing temperatures reaching up to 2.5°C. In contrast, in the absence of inhomogeneity, the temperature drop in these points approaches near 0°C, indicating the adverse effect of inhomogeneity on freezing near the pipe when heterogeneity is present. The primary cause of this can be attributed to the lower thermal conductivity of the inhomogeneity mass compared to the surrounding soil, meaning that at a given time, the inhomogeneity allows freezing to develop beyond its region at a slower rate. At a distance of 3 meters from the freezing pipe (RR=3), the temperature variations match those of the no- inhomogeneity case up to time step 300. However, beyond this step, the presence of inhomogeneity causes the temperature to converge to a constant value of approximately 5°C. Without inhomogeneity, the temperature drop is more optimal, with values even below 5°C. This is justifiable because subsurface inhomogeneity, due to its lower thermal conductivity, requires a longer time than the surrounding soil to reach freezing temperature. Therefore, during the same period that the freezing radius is developing in the soil, the inhomogeneity remains at higher temperatures, and heat flux flows from the inhomogeneity toward the surrounding soil, which is now at a lower temperature. This effect is more pronounced at a distance of 6 meters from the freezing pipe (RR=6), where conditions are identical only up to time step 200. With the inhomogeneity distance increased to W=0.25L in Figure (3-b), near the freezing pipe (RR=0), the effects of inhomogeneity are reduced by 10% compared to the previous case, and the presence of inhomogeneity shows a lesser impact on the final freezing value. This effect is also noticeable at evaluation points 3 meters (RR=3) and 6 meters (RR=6) away.

Figure (4) presents temperature variations along the centerline of the freezing pipe at distances of 0.1, 3, 6, and 9 meters from the freezing pipe in the presence of a circular subsurface inhomogeneity. Figure (4-a) shows the variation results for the case where the distance of the inhomogeneity from the freezing pipe is set to W=0.12L. According to this figure, at points close to the freezing pipe (RR=0), the circular inhomogeneity causes the final temperature to converge to higher values around 4°C, whereas in the absence of inhomogeneity, the temperature in these points approaches near 0°C. Compared to the square-shaped, it can be observed that the circular shape has a more adverse effect on freezing development, as in the square inhomogeneity case, the temperature reaches lower values around 2.5°C, which is approximately 40% lower than the final temperature with a circular cross-section inhomogeneity. This behavior is also observable at other evaluation points at distances of 3, 6, and 9 meters. The more adverse effect of circular inhomogeneity compared to square shape on freezing development can be justified by the fact that the second-order geometry of a circle distributes temperature uniformly in all directions. At a given time, as the freezing zone radius develops around the freezing pipe, due to its lower thermal conductivity, the inhomogeneity acts as a heat source, and its circular shape distributes heat uniformly to the surrounding areas.

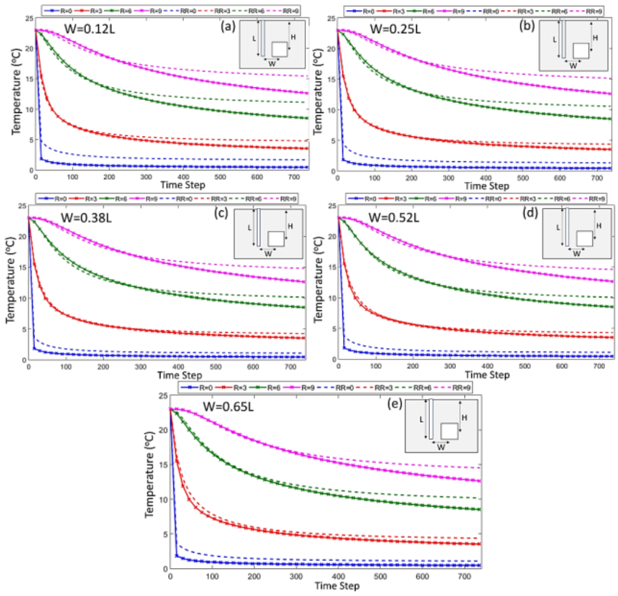

Figure 3. Comparison of temperature variations along the freezing pipe axis at different distances when a square inhomogeneity is located at distances of: a) W=0.12L, b) W=0.25L, c) W=0.38L, d) W=0.52L, and e) W=0.65L from the freezing pipe. In this figure, the curves labeled R=0 to R=9 correspond to temperature variations in the absence of heterogeneity, while the curves labeled RR=0 to RR=9 correspond to temperature variations in the presence of inhomogeneity.

In contrast, in sharp corner geometries, heat distribution is maximal at the centers of the sides and minimal at the corners. Therefore, heat distributes more effectively around rounded-corner inhomogeneity, further reducing the quality of artificial ground freezing. By increasing the inhomogeneity distance to W=0.25L in Figure (4-b), it can be seen that the effect of inhomogeneity on temperature variations at points 0.1 and 3 meters away is reduced. In Figures (4-c) to (4-e), it can be observed that the effects of inhomogeneity persist even when its distance from the freezing pipe reaches W=0.65L, affecting the results at distances of 3 to 9 meters, with its intensity decreasing at a slow rate.

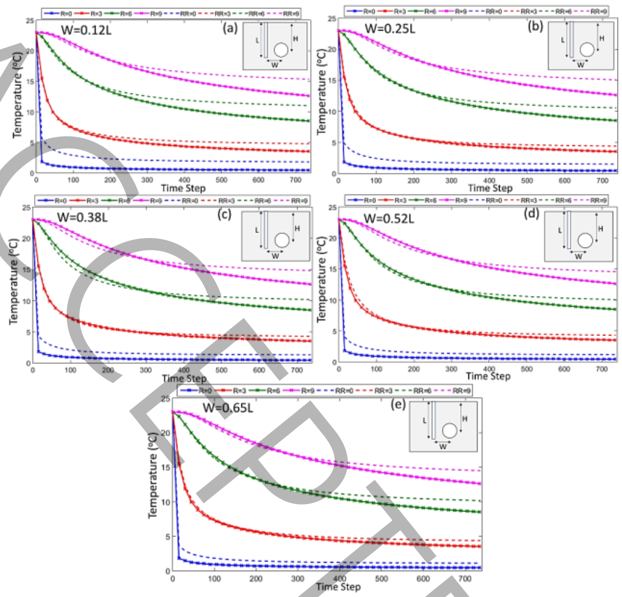

Figure 4. Comparison of temperature variations along the freezing pipe axis at different distances when a circular inhomogeneity is located at distances of: a) W=0.12L, b) W=0.25L, c) W=0.38L, d) W=0.52L, and e) W=0.65L from the freezing pipe. In this figure, the curves labeled R=0 to R=9 correspond to temperature variations in the absence of heterogeneity, while the curves labeled RR=0 to RR=9 correspond to temperature variations in the presence of inhomogeneity.

5. Conclusions

The results of this research indicate that the hybrid method provides an optimal approach for modeling the artificial ground freezing phenomenon in soil and is capable of being applied to solve various problems involving complex geometries and subsurface heterogeneities. Ultimately, the findings of parametric studies can be summarized as follows:

- The geometry of artificial ground freezing development takes the form of a bubble, which narrows at the top due to heat exchange with the ground surface boundary, while having the greatest volumetric influence at the center. This observation aligns well with results from previous laboratory studies and numerical modeling.

- The presence of subsurface inhomogeneity, in the form of an unsaturated rock mass, reduces the effective radius of artificial ground freezing in the soil. This can be justified because the low thermal conductivity of the unsaturated rock mass creates a region with positive heat flux from the rock mass toward the surrounding soil, thereby reducing the freezing zone radius.

- In the presence of square-shaped inhomogeneity, the influence radius decreases by an average of 15%, while for circular-shaped inhomogeneity, it decreases by up to 20%.

- According to the research findings, the minimum distance between the freezing pipe and the inhomogeneity that has the least effect on the freezing influence radius is estimated to be W=0.52L (where L is the length of the freezing pipe).

- Circular-shaped inhomogeneity demonstrates more adverse effects on freezing development around the freezing pipe compared to square-shaped inhomogeneity, indicating the negative impact of rounded-corner rock masses on the quality of ground freezing.

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