Damage identification in conical shell structures using analytical methods based on free vibration of the structure

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ABSTRACT

Structural health monitoring and detection of damaged areas in a structure is essential for surveying its performance during the structural life. Because not paying attention to the local damages is reduced the efficiency of the structural performance and could result in increasing the damaged intensity due to natural or artificial loads and consequently collapse of entire structure. The main goal of this research is to propose a model using analytical methods and developing it to identify the damaged areas in conical shells made of functionally graded materials placed on an elastic substrate. Since the mode shapes are sensitive parameters to damage presence in the elements, they are used for damage detection process of the structure. The governing equations have been extracted based on the first-order shear theory in Maple software. The analytical method uses the power series to obtain the natural frequencies and mode shapes of the structure. In order to ensure the performance of the proposed model, the results of the modeling have been verified with the results of other references and those of the numerical model built in Abaqus software. Finally, taking into account various types of support conditions and different stiffnesses for the substrate, the damaged elements of the conical shell have been specified considering different damage scenarios. Investigations show the proper performance of the proposed model in detecting the damaged area.

KEYWORDS

Functionally graded materials (FGM), Conical shell, Damage identification, First-order shear deformation theory, Modal derivatives shapes.

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1. Introduction

Identification of local damages in structures is of particular importance. Because ignoring them reduces the efficiency of the structure and may be aggravated by loading from natural or artificial hazards and ultimately lead to the destruction of the entire structure. The effects of local damage to the structure are changes in the strength and stiffness at that location. These changes affect the static and dynamic behavior of the structure, which leads to changes in its dynamic and static responses, including its natural frequencies and mode shapes. As a result, by having the modal information and its derivatives in the healthy and damaged states of the structure, it is possible to check the location of the damage. In 2016, Alipour presented a new approach for the static analysis of circular sandwich plates based on layering, which is very similar to the three-dimensional elastic theory. Each layer of the sandwich plate is made of Functionally Graded Material (FGM). The boundary conditions can be on the elastic bed, which is assumed to be non-uniform normal stress or tensile shear. The transverse shear, normal stress, and boundary conditions at the top and bottom of the sandwich plate, as well as between the layers, are investigated[1].

In 2019, Mohammadezazadeh and Jafari studied the active vibration behavior of a composite layer of simple rotating cylindrical shells confined in a magnetic layer using the first-order shear deformation theory. In this study, the details of the influence of various parameters on the damping characteristics and response of the vibration behavior were investigated[2] In 2020, Agha Ahmadi et al. studied the free vibration behavior and also identified the damaged area of the cylindrical shell structure[3].

In 2024, Fan and Lezgi [4] investigated an advanced and efficient shell theory with only seven displacement field variables. In this study, an eight-node quadrilateral shell element was used to solve the dynamic equations of motion. The accuracy of the proposed shell theory and the developed finite element formulation were evaluated through free vibration analysis of cross-laminated, angled, and soft-core sandwich multilayer shells. In 2024, Yao and Lezgi[5] presented a new theory based on the shear deformation of the shell with seven unknown parameters, and the governing equations of the shells under static loads were solved by developing an eight-node shell element. Also, the kinematic relations presented for the multilayer shell model were expressed based on the double superposition derivative.

As can be seen in previous studies, research conducted on conical shells has focused on the modeling process and has not addressed the health monitoring and damage caused in such structure. Also, studies have been conducted on determining the location of damage on other structures such as beams and flat plates, etc. In this research, analytical modeling has been done on conical shells with FGM materials and the presence of damage and its location have been identified. First, the governing equations of the structure were extracted based on the first-order shear theory in Maple software, and then the mode shapes and natural frequencies were obtained using the power series method. The model was validated by comparing the results with those of other references and the numerical model built in Abaqus software. The modeling process and the detection of the damaged location of the shape were evaluated by considering different damage scenarios in different support conditions and different stiffnesses for the bed.

2. Governing equations for a FGM conical shell

Figure (1) shows the geometric characteristics of the conical shell.

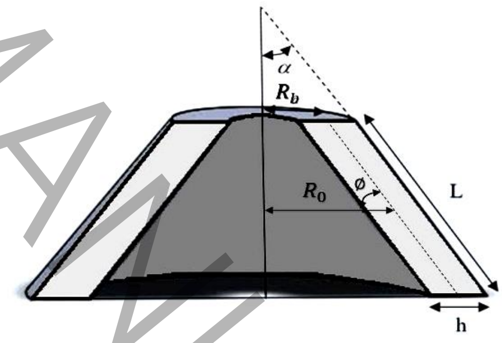


Figure 1. Geometry of incomplete conical

The equations of motion may be calculated using the principle of potential energy minimization.

$$\delta U + \delta K + \delta V = 0 \tag{1}$$

where U is the strain energy, K is the kinetic energy, and V is the energy due to the external load. Applying this principle leads to three equations of motion for the conical shell:

$$\delta K = -\int_V \rho \left\{ \ddot{U} \delta U + \ddot{W} \delta W \right\} dV, \tag{2}$$

$$\delta U=\int_V\left\{\sigma_x\delta\left(u_{0,x}+z\psi_{x,x}\right)+\sigma_\theta\delta\left(\frac{w}{R_0}\right)+\tau_{xz}\delta\left(\psi_x+w_{,x}\right)\right\}dV$$

$$\delta V=\int_V\left[k_w(\gamma_0+\gamma_1 x+\gamma_2 x^2)w\delta w\right]dA$$

Different types of boundary conditions for the conical shell are expressed as follows:

$$\begin{array}{ll} \text{Simple support:} & U_x=0, M_x=0, w_x=0, \\ \text{Free end:} & N_x=0, M_x=0, Q_x=0, \\ \text{Fixed end:} & u_x=0, \psi_x=0, w_x=0, \end{array} \tag{3}$$

Taylor power series is used to solve the equations, and the unknowns are presented based on Taylor power series as follows:

$$\begin{aligned} u_x &= \sum_{i=0}^{\infty} U_i(x-x_0)^i \\ \psi_x &= \sum_{i=0}^{\infty} \Psi_i^{(x)}(x-x_0)^i \\ w_x &= \sum_{i=0}^{\infty} W_i(x-x_0)^i \end{aligned} \tag{4}$$

By using Taylor's power series and by inserting the relevant equations, the governing equations and boundary conditions can be expressed as a series, respectively. The number of initial boundary conditions of the cone is 3 relations, the final boundary conditions of the cone are 3 relations, and if the conical shell consists of several parts, the continuity conditions at the junction of each part are 6 relations. In total, there are 6n relationships for the boundary conditions and the continuity of the cone, where n is the number of sections along the length of the cone. These relations can be expressed as matrix form. Therefore, by taking the determinant of the matrix, the natural frequency for the conical shell is obtained.

3.Verification of the numerical model of the conical shell

To verify the accuracy of the proposed model used, the results obtained from the proposed model are compared with the results in reference[6] and also with the results of numerical modeling in Abaqus software. The specifications of the conical shell materials used in the validation study are presented in Table (1).

Table 1- Characteristics of conical shell materials

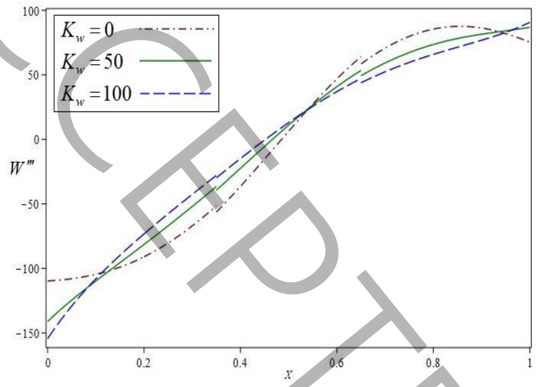
Density (kg/m^3)	Modulus of elasticity (GPa)	Poisson's ratio
5700	168	0.3

In order to model in Abaqus software, three-dimensional elasticity theory is used in axisymmetric mode. For modeling, 2500 axially symmetric 8-node elements (CAX8R) have been used. The number of elements has been determined based on sensitivity analysis. in such a way that by reducing the dimensions of the element, there has been no significant change in the response. The results obtained from the proposed model are in good agreement with the results in reference[6] and Abaqus software. It is worth noting that the finite element method was used in reference[6] and Abaqus software, so the results obtained are very close to each other. Also, the results of the proposed model for the FGM conical shell are in acceptable agreement with the results of Abaqus software.

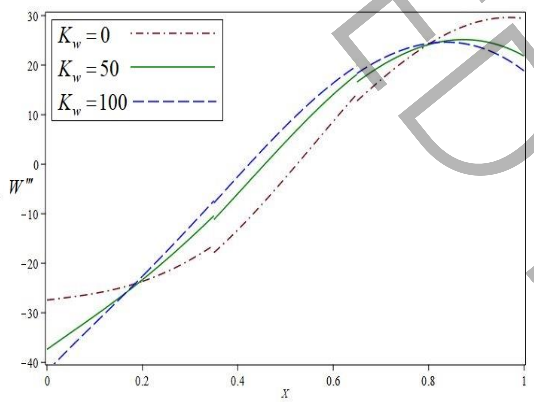
4.Investigating the performance of the proposed numerical model at damage locations with different scenarios

In the following study, damage was created in different layers of a FGM conical shell and the dimensionless frequency of the first mode of undamaged and damaged structures was presented for different support conditions and for different stiffness coefficients. The corresponding damage has been applied through a reduction in the modulus of elasticity of the materials of the layers. The damage scenarios are selected in such a way that the accuracy of the model for single, double and multiple failures as well as the proximity of damaged elements with different error percentages is examined. By examining the results, it is observed that the frequency of the first mode of the damaged structure decreases compared to the frequency of the undamaged mode in all scenarios, and the frequency increases with the increase in the stiffness coefficient of the elastic bed, which increases the rigidity of the bed. Therefore, the changes of this parameter for the first mode shape of the damaged structure in different damage scenarios are examined. The following figure shows the changes in the third derivative of the first mode shape along the cone for the damaged structure for each of the scenarios. As can be seen, the third derivative of the displacement for the damaged structure is shown in the third element with stiffness coefficients of 0, 50 and 100; as can be seen, the third derivative has a discontinuity at the beginning and end of the third element (at x=0.35m and x=0.65m relative to the beginning of the cone), which indicates damage in this element, and in other parts, the diagram is continuous, indicating the absence of damage in other elements of the structure. It is also observed that with a decrease in the bed coefficient, the discontinuity at the desired points is more noticeable. The above cases are valid for both fixed and simple support cases.

According to what has been shown, the proposed method for modeling FGM conical shells will be able to detect the extent of damage and can be used in real structures. The proposed method could be extended to real structures.



(a)



(b)

Figure 2. Third derivative values of the first mode frequency for scenario (1); a. for fixed supports; b. for simple supports

5.Conclusion and summary

In this study, an analytical method based on first-order shear theory for modeling FGM conical shells has been proposed. By extracting the governing equations and in order to verify the accuracy of numerical modeling, the obtained results have been compared with the results presented in reference[6] and also with the results obtained from Abaqus software for FGM conical shells with simple two-end supports, two-end fixed supports and two-end free supports, which had an acceptable agreement in both validation cases. In the following, to check the efficiency of the proposed method in detecting the damaged area of the structure, the conical shell was divided into five elements; single, double and multiple failure scenarios were defined under different support conditions and by applying different bed conditions. With the help of derivatives of mode shapes, the damage area was investigated. The investigations indicate the appropriate performance of the proposed model in detecting the damage location.

6. References

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