Analysis of Dynamic Stability of Frame Columns under The Effect of Concentrated Mass and Inherent Damping by Finite Element Method

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ABSTRACT

Analysis of stability in columns as the main structural member has a special place in engineering research. In most of the past research, generally, researchers have studied the static buckling in columns (prismatic or non-prismatic) in (building frames or industrial beams). Static load capacity only expresses the static critical load capacity of members under gravity load. For the safe design of the structure, it is necessary to check the dynamic stability of the columns in the building frames under the vertical load of an earthquake. In this article, in a comprehensive model, the combined effect of inherent damping, floor mass and vertical earthquake load on the dynamic stability of columns in unrestrained moment frames is investigated. In fact, the proposed method is a combination of Julian-Lawrence static modeling and Bolotin dynamic modeling to consider the dynamic effects in the frame columns based on the finite element method. In the first step, the constitutive equation is extracted using Hamilton's method. In the next step, the response of the equation is checked using the finite element method with Hermitian three-degree interpolation functions for 50 components. The results show that the inherent damping, concentrated mass and rotational stiffness of semi-rigid joints have a significant effect on the resonance frequency, effective length and dimensionless dynamic load factor. With the increase in inherent damping, rotational stiffness and concentrated mass, the graph of effective length changes is shifted to the left side of the excitation frequency axis. Considering the effects of inherent damping and concentrated mass in the modeling, 7% and 81%, respectively, affect the resonance frequency changes. There is an acceptable agreement between the results of the present article and previous research.

Keywords: Dynamic Buckling, Dynamic Stability, Effective Length, Critical Load Capacity, Analysis Eigenvalue

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1.Introduction

Structural stability has always played a vital role in the design of steel structures, particularly for slender members where buckling becomes a governing factor. While Euler's classical theory laid the foundation for defining the critical buckling load, it was limited to ideal boundary conditions [1]. More realistic approaches later incorporated semi-rigid beam-to-column connections and elastic supports through equivalent spring models [2]. Further developments included creep buckling in viscoelastic columns modeled via Kelvin and Maxwell frameworks [3], buckling of various structural elements including tapered or stepped columns [4], and columns with variable cross-sections under different boundary conditions [5]. Analytical solutions using Hermite shape functions, virtual work principle, and energy-based methods provided accurate predictions for critical loads and natural frequencies [6]. Recent studies have extended to dynamic stability of columns under time-dependent axial loads. Bolotin pioneered the concept of dynamic buckling in mechanical systems [7]. Later work included studies on graded nanobeams, viscoelastic fractional-order beams, and steel frames with semi-rigid joints under harmonic excitation [8]. In 2023, Savin et al. investigated progressive collapse in RC frames under sudden column removal scenarios [9]. Kadim and Alzuaid validated FEM models for tapered RC columns, confirming improved load capacity [10]. Ozil et al. analyzed the dynamic stability of diamond-shaped steel frames and found geometric form enhances resistance to external oscillations [11]. Fonseca (2024) revealed that non-uniform square hollow steel columns are more resilient to initial buckling than uniform ones [12]. Despite these contributions, dynamic buckling under vertical seismic loading remains underexplored in design codes such as Iran's National Building Regulations [13]. Most prior research also overlooks the combined effect of inherent damping and floor mass concentration on column stability in unbraced steel frames. This research develops a unified model for analyzing dynamic buckling, effective length, and natural frequency of prismatic columns in unbraced steel moment frames, incorporating:

- Vertical seismic effects modeled as harmonic axial loads,
- Inherent material damping using Rayleigh's method [14],
- Semi-rigid beam-to-column joints via rotational springs,
- Elastic boundary conditions.

The governing equations are derived using Hamilton's principle and energy method, with Hermite interpolation functions used to form the stiffness, geometric, mass, and damping matrices. The static buckling load and natural frequency are obtained through eigenvalue analysis. Finally, the dynamic critical load under varying excitation frequencies is determined using the Müller root-finding algorithm, with validation against prior literature.

2. Methodology

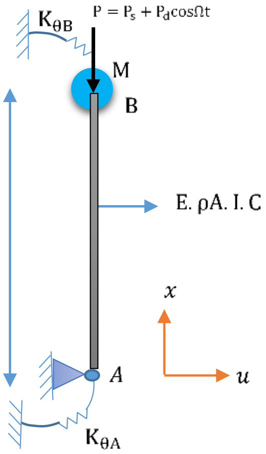

Figure 1: Column in an Unbraced Moment Frame Considering Lumped Floor Mass and Inherent Damping Effects

As illustrated in Figure 1, a column belonging to an *unbraced moment-resisting frame* is considered, characterized by the moment of inertia I, modulus of elasticity E, cross-sectional area A, material density ρ, and length L. The column has a mass per unit length of ρA. The effect of inherent (material) damping is modeled using a damping coefficient C, while the inertial effect of the floor mass is represented as a lumped mass M applied at the top of the column. The column is assumed to be part of an unbraced steel moment frame, where point B (the top of the column) is free to undergo lateral displacement, i.e., has no lateral stiffness. The semi-rigid behavior of beam-to-column connections at the column ends (points A and B) is modeled using rotational springs with stiffness values KθA and KθB respectively. During seismic events, columns in such frames are subjected to dynamic loads induced by both *lateral and vertical components* of earthquake excitation. To incorporate the effects of vertical seismic acceleration, the column is loaded axially with a time-dependent axial force P(t), which is composed of a harmonic cosine component superimposed on a constant static axial load. This study analyzes the dynamic buckling of a prismatic column in an unbraced steel moment frame using the Finite Element Method (FEM). The modeling framework integrates:

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- Inherent damping via the Rayleigh damping model
- Concentrated mass (M) representing the floor mass
- Rotational stiffness at ends using semi-rigid connections
- Dynamic harmonic axial loading to simulate vertical seismic action

2.1. Governing Equation

The governing differential equation is derived using Hamilton's principle. The total virtual work includes kinetic energy, strain energy, work by damping, concentrated mass, and boundary springs:

$$\delta\int_{t_0}^{t_1}(T - U + W_d + W_s)dt = 0 \quad (1)$$

The kinetic energy incorporates both distributed mass mmm and lumped mass MMM:

$$T = \frac{1}{2}\int_0^L m\left(\frac{\partial u}{\partial t}\right)^2 dx + \frac{1}{2}M\left(\frac{du(L,t)}{dt}\right)^2 \quad (2)$$

The strain energy includes the bending stiffness and axial load effects:

$$U = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 u}{\partial x^2}\right)^2 dx - \int_0^L P(t)\left(\frac{\partial u}{\partial x}\right)^2 dx \quad (3)$$

The damping is modeled as:

$$C = \alpha M + \beta K \quad (4)$$

where α, β are Rayleigh coefficients.

Using Hermite cubic interpolation functions and Galerkin's method, the weak form is discretized and assembled into a global eigenvalue problem:

$$(K + K_g + \omega C - \omega^2 M)U = 0 \quad (5)$$

Solving this yields the resonance frequencies and dynamic buckling loads as functions of damping, mass, and joint stiffness.

3. Results and Discussion

3.1. Effect of Boundary Stiffness and Mass

The analysis was conducted with varying parameters to study their effect on dynamic buckling. The results confirm:

- Increased concentrated mass significantly decreases the resonance frequency.
- Higher damping shifts the resonance frequency leftward, reducing peak load capacity.
- Rotational stiffness in semi-rigid joints improves dynamic stability.

3.2. Parametric Study Summary

Table1.Parametric Study Summary

Parameter	Change in Resonance Frequency
Damping increase (ζ = 4%)	↓ 7%
Mass increase (to 3 Ton)	↓ 81%
Stiffness increase (Ga, Gb)	↑ 13%

3.3. Numerical Convergence

The FEM model showed convergence for ≥ 20 elements, with minimal error compared to reference solutions (Julian & Lawrence, 1980; Bolotin, 1962). The study validated results using analytical formulas and numerical benchmarks across ideal and realistic boundary conditions.

3.4. Sensitivity Analysis of Model Parameters on the Dynamic Response of the Column

This section evaluates the influence of three key parameters—intrinsic damping, concentrated floor mass, and boundary stiffness of connections—on the dynamic stability of the column. A normalized sensitivity index was calculated for each parameter to quantify its effect on the column's resonance frequency. The results show:

- Concentrated floor mass has the greatest negative impact, significantly reducing the resonance frequency and dynamic stability.
- Increasing boundary stiffness raises the resonance frequency, enhancing stability.
- Intrinsic damping has a relatively smaller effect, causing a slight decrease in resonance frequency.

These findings provide engineers with a clear prioritization for seismic retrofitting and earthquake-resistant design.

Table2. Normalized Sensitivity Index Table

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Input Parameter	Relative Change (ΔP/P)	Response Change (Δf/f)	Sensitivity Index S
Intrinsic Damping ζ	1	-0.068	-0.068
Concentrated Floor Mass M	1	-0.812	-0.812
Boundary Stiffness Ga,Gb	1	+0.133	+0.133

4.Conclusion

This study investigates the dynamic buckling, natural frequency, and effective length corresponding to the dynamic buckling of an unbraced column with elastic supports, considering intrinsic damping and concentrated mass (due to floor mass) under harmonic axial loading. Initially, the weak form of the governing differential equation was derived. Interpolation functions were employed as shape functions in the formulation, based on which the material stiffness, geometric stiffness, and mass matrices were obtained. Subsequently, the eigenvalue problem was solved using these stiffness matrices. The Müller root-finding technique was implemented via MATLAB coding to calculate the eigenvalues.

Key findings of the study are as follows:

- The intrinsic damping coefficient significantly influences the variation of the effective length associated with dynamic buckling as a function of the excitation frequency. Increasing damping shifts the response curve to lower excitation frequencies.
- The parameters GA and GB representing the stiffness ratios of the column to beam at the semi-rigid connections at the base and top respectively, also considerably affect the effective length variation related to dynamic buckling versus excitation frequency. Increasing these stiffness ratios causes the response curve to shift towards lower frequencies.
- Incorporating intrinsic damping in the equations affects the resonance frequency by up to 7% compared to the undamped model.
- The concentrated mass (resulting from floor mass) has a pronounced impact on the effective length variation with respect to excitation frequency. Increasing the concentrated mass shifts the curve to lower frequencies, with its effect on the resonance frequency reaching up to 81%.

5.References

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