Linear Programming and Moving Morphable Components Approach in 2D Structural Topology Optimization

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ABSTRACT

Moving morphable components (MMC) is a relatively new and effective approach in structural topology optimization. In comparison with other common methods in topology optimization such as density-based methods and level-set-based methods, it requires fewer design variables and the boundary of the structure is defined explicitly. However, the obtained topology is highly dependent on the initial shape and position of the components. On the other hand, plastic layout optimization utilizes linear programming to find the global optimum of the structural optimization problem. Assuming rigid plastic behaviour for material, the optimum layout can be obtained quickly and accurately. The optimum layout gives only the area of members which is constant along the members, therefore, there is no detail about the connection between members. Hence, the obtained optimum layout cannot be used directly for manufacturing methods such as additive manufacturing. It can be shown that the minimum compliance optimization problem for a single load case is equivalent to a minimum-weight plastic layout optimization formulation. This study utilizes the idea and presents a two-step method to take advantage of and compensate for the shortcomings of these two methods in the topology optimization of 2D structures. To this end, in the first step, the optimum layout is obtained using linear formulation in layout optimization and then, the obtained layout is utilized as an initial point in the MMC approach. The results show the efficiency, accuracy, and high convergence rate of the proposed method.

KEYWORDS

Topology optimization, Layout optimization, Linear programming, Moving morphable components, Additive manufacturing.

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1. Introduction

This study presents a new approach to overcome the complexities associated with non-convex optimization in continuum-based topology optimization, a field pivotal to achieving maximum structural efficiency. Non-convex optimization poses significant challenges in locating global optima, particularly in engineering applications where precise material allocation is critical. Heuristic methods, such as genetic algorithms, are commonly employed to address these difficulties. However, while these techniques improve the chances of identifying optimal solutions, they often come with the drawback of high computational demands, especially when handling a large number of design variables [1]. To address these limitations, this research introduces a two-phase framework for topology optimization. The initial phase utilizes Layout Optimization (LO), to generate a preliminary design through well-established mathematical principles. This phase is rooted in Michell's 1904 criterion for optimal truss structures, a foundational concept that has significantly influenced structural optimization [2]. Initially underutilized, Michell’s criterion gained traction with later adaptations, such as Dorn’s 1964 approach leveraging linear programming to refine structural design [3]. Over time, advancements in LO have incorporated solutions for diverse loading conditions and self-weight considerations, exemplified by innovations like the member-adding algorithm proposed by Gilbert and colleagues, which enhances the overall efficiency of designs [4, 5]. Figure 1 visualizes the phases of the layout optimization process, including the initial design domain, discretization into nodes, the ground structure framework, and the final optimized design.

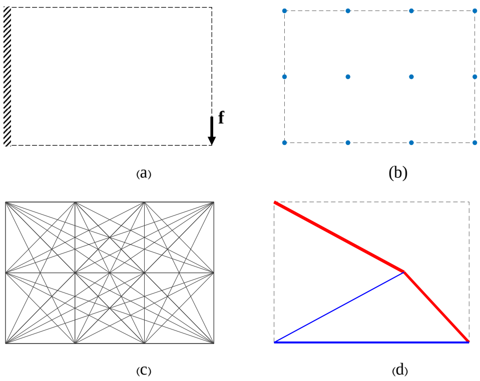

Fig. 1. (a) Design domain, (b) Discretization of the design domain with nodes, (c) The ground structure, and (d) The optimal design

The second phase of the study leverages the innovative Moving Morphable Components (MMC) method, a cutting-edge technique designed to optimize the positioning and configuration of a finite number of structural components to establish the final topology. This approach utilizes a mathematically defined scalar function, streamlining the design process with enhanced efficiency. By refining the geometry and location of these components, the MMC method effectively integrates aspects of shape and topology optimization. This integration enables direct enforcement of constraints, such as element size limitations or buckling resistance, which are traditionally difficult to address in conventional topology optimization.

One of the key advantages of the MMC method is its reliance on a reduced number of design variables compared to other methodologies, resulting in accelerated optimization workflows. Nevertheless, it faces difficulties due to its intrinsic non-linear and non-convex characteristics. The success of the MMC approach heavily depends on the initial design, which plays a critical role in guiding the optimization process and shaping the final results. Fig. 2 illustrates the MMC approach as the second phase, showcasing the transformation from the initial component layout to the optimized 2D structural topology [6-11]

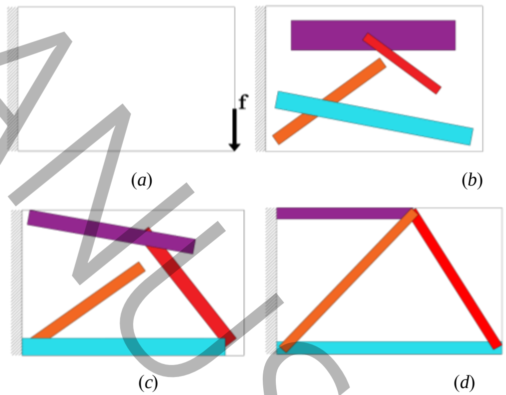

Fig. 2. (a) design domain and boundary conditions, (b) initial state of components, (c) movement of components to achieve the optimal topology, and (d) final optimal topology.

The combination of these two phases aims to improve convergence speed and overall design quality, ultimately leading to more effective structural optimization outcomes.

2. Methodology

This study implements a two-phase optimization process to enhance the MMC approach in topology optimization. The first phase employs LO, which formulates a convex optimization problem to generate an optimal initial design using linear programming techniques. This phase builds on established theories, ensuring that the LO solution minimizes strain energy and provides a robust starting point. The MOSEK solver is specifically used to solve linear programming. In the second phase, the MMC method utilizes the LO-generated design as its initial layout, significantly mitigating the risk of converging to local optima. The transition from LO to MMC involves transformation equations that link the initial design to the MMC framework. Additionally, the design space is constrained by limiting variations in angles, lengths, and thicknesses, which reduces the number of design variables and accelerates the convergence of the optimization process. To solve the non-convex optimization problem, the Method of Moving Asymptotes (MMA) is employed, with both phases implemented in MATLAB.

3. Results and discussion

The proposed two-phase optimization approach was evaluated through three numerical examples: a cantilever beam, a simply supported beam, and an L-shaped structure, demonstrating its effectiveness in enhancing structural performance while reducing computational time compared to the conventional MMC method. In the first example, the cantilever beam, subjected to a unit load, achieved an objective function value of 128.8 in 589 seconds using the two-phase approach, compared to 129.6 in 986 seconds with conventional MMC. The second example, involving a simply supported beam, yielded a final value of 210.9 in 587 seconds for the two-phase method, while the conventional approach took 3482 seconds for a value of 237.8. In the third example, the L-shaped structure reached an optimal solution of 155.0 in just 32 seconds, significantly faster than the conventional method, which took 1937 seconds for a value of 172.2. Overall, the two-phase methodology consistently outperformed the conventional MMC method, highlighting its robustness and potential for broader applications in structural optimization.

4. Conclusion

This research introduces a two-phase structural optimization approach to enhance the convergence speed and outcomes of the Moving Morphable Components method, addressing the challenges posed by its non-convex nature. In the first phase, an optimal structural layout is obtained through layout optimization, which is computationally efficient and serves as a robust starting point for the MMC method. The effectiveness of this approach is validated through numerical examples, showing significant improvements in both convergence speed and optimality compared to conventional MMC methods. Results indicate that the two-phase approach consistently yields better objective function values and reduced optimization computational times. The observed rapid convergence is attributed to starting near the optimal solution and applying tighter constraints on specific design variables, resulting in more manufacturable topologies and less need for post-processing.

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