Two-dimensional dynamic analysis of rectangular tanks under the effect of harmonic and seismic loading by method of fundamental solution with pressure formulation

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ABSTRACT

It's important to study liquid motion and its effect on the tanks. The method of fundamental solution (MFS) is a novel meshless numerical method proposed to solve incompressible inviscid fluid flow problem with moving boundaries. In this paper, this method is developed for two-dimensional rectangular water reservoirs under harmonic and earthquake excitations. For modeling of fluid motion with moving free surface, Lagrangian formulation is used to pressure equation, like a potential equation and so the geometry is updated in each time step through an implicit algorithm. In recent research, equations are used with linearized boundary conditions, while due to Lagrangian approach of pressure-based equations; the boundary conditions of the problem are very simple and it's easy to solve complex problems. Innovation of this study is considering earthquake loads to simulate sloshing water surface applied by Method of fundamental solution (MFS). The nature of earthquake excitation due to frequency content and fast acceleration changes lead to singularity problem in tank corners. So, the solution is expressed as a linear Green basis functions in method of fundamental solutions to avoid the singularity problem and to obtain better results. The numerical results are compared with other numerical and experimental results to show proposed procedure precisely taking into account the effects of earthquake excitation.

KEYWORDS

Rectangular tanks, pressure formulation, method of fundamental solutions (MFS), Lagrangian algorithms, harmonic and seismic excitations

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1. Introduction

Water tanks are considered as special structures and therefore it's important to analysis of water behavior inside the tank and calculation of hydrodynamic forces on the tank walls. Hoskins and Jacobson [1] provided the first analytical and experimental results of rigid rectangular tanks which is given an arbitrary small oscillation. Housner [2] developed the rectangular and cylindrical tanks with a two degree of freedom model taking into account the convective mass and impulsive mass of water by simplified dynamic analysis. This model has been used in most of current codes and standards.

Heavy tank damages cause by the earthquake persuade scientists to consider sloshing water. Sloshing means any motion of a free liquid surface inside the tank. Tang and Veletsos [3] studied the effect of flexible cylindrical tanks on the magnitude and distribution of hydrodynamic pressures and also it's carried out for calculating the deflection of tank walls by application of the Rayleigh Ritz method. It's shown that if the effect of surface waves is neglected, the solution is reduced to the impulsive effect only. Chen and Kianoush [4] proposed combining the added mass and sequential methods for computing hydrodynamic pressures in 2D rectangular tank in which the effect of flexibility of the tank wall is taken into account. They found that with the increase in the flexibility of the tank walls and decrease in the liquid level in the tank, the natural frequencies of the liquid tank increase. Also they concluded that the base shear and acceleration of the reservoir have a linear relationship with wall flexibility. In this study, a shallow tank and a tall tank are used to investigate of the effects of liquid level and width of the tank on the results.

Chen et al [5] study on the sloshing behaviors of cylindrical and rectangular liquid tanks subjected to harmonic and recorded earthquake excitations. A 3D boundary element method (BEM) for space established to simulate the sloshing phenomenon with the second-order Taylor series used to update the geometry in time.

Shekari [6] scrutinize the sloshing response for multi baffled flexible cylindrical container subjected to lateral seismic excitations. In this study, sloshing natural frequencies and seismic response compute with developed implicit algorithm of coupled numerical criterion based on the FE discretization for the tank wall and BE discretization for the fluid region. He concluded that the installation of baffle rings inside the liquid tank can be significantly diminished the dynamic response during severe earthquake ground shakings such as sloshing response, hydrodynamic pressure and shell radial displacement. Furthermore, owing to the high contribution of impulsive pressure and a time mismatch between peak impulsive and convective response, the position and width of baffle is more influential on seismic behavior of the tank.

Zandi et al [7] investigated the fluid motion with moving boundaries in rigid rectangular tank by exponential basis functions (EBFs). This basis function in the domain can be locally or global approach. In the local EBFs method, there are some cloud of nodes construction on each nodal points and continuity between the local solution in the adjacent clouds to solve governing equation. If the bases are defined globally on the whole solution domain, all nodal points are involved in computation for one node in the region at each step time. In this study, the Lagrangian formulation is used to update geometry in time.

2. Methodology

The governing equations for the Newtonian, incompressible and non-viscous fluid flow include conservation of mass and momentum equations, which are called Navier-Stokes equations. By satisfy appropriate boundary conditions, the following governing equations are obtained.

$$\nabla^2 p = 0 \tag{1}$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \rho \mathbf{g} \tag{2}$$

Where in (1) and (2) p is pressure, $\mathbf{u}$ is the vector containing the Cartesian component of the velocity field, ρ is the density and the vector $\mathbf{g} = < 0 \quad -g >^T$ is used to define the source term vector which includes the gravity acceleration g. Also, D/Dt denotes the total and material derivative of the quantity and ∇ is the wall known gradient operator. We assume pressure as

$$p = p_H - \rho g y \tag{3}$$

Where y is the vertical coordinate, introducing (3) in (1) and (2) results in the following equation.

$$\nabla^2 p_H = 0 \tag{4}$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p_H \tag{5}$$

If the solution boundaries Γ are considered as a Neumann boundary condition on the slip impermeable boundaries in contact with fluid-structure interfaces (Γ_S) and a Dirichlet boundary condition on the free surface of fluid (Γ_F), the boundary conditions for the problem lead to the following relations.

$$p_H = \rho g y \quad on\ \Gamma_F \tag{6}$$

$$\frac{\partial p_H}{\partial n} = -\rho \mathbf{n}^T \mathbf{a}_s \quad on\ \Gamma_S \tag{7}$$

3. Results and discussion

We consider the problem of water sloshing in a rectangular rigid tank under earthquake excitations and comparing it with laboratory results.

3.1 Numerical Example: rectangular water tanks

water tank with the length $L = 0.8\,\text{m}$, the width $B = 0.141\,\text{m}$, and the still water depth $h = 0.1\,\text{m}$ is studied in this example subjected to 0.01 Chi-Chi earthquake excitation and the results have been compared with numerical and experimental data. The water density and gravity acceleration are $\rho = 1000\,\text{kg/m}^3$ and $g = 9.81\,\text{m/s}^2$. Thus, the first natural frequency ω_1 based on linear wave theorem, can be calculated as follows

$$\omega_1 = \sqrt{\frac{\pi g}{L}\tanh\frac{\pi h}{L}} \tag{8}$$

Thus, the first natural frequency of this tank is $\omega_f = 3.79\,\text{rad/s}$. For dynamic analysis, ChiChi record in 1999 is used with PGA (peak ground acceleration) of the input acceleration time history is scaled to $\text{PGA} = 0.01 \times 0.258g$ according to reference [5]. Figure 1 shows the water elevations on the right lateral wall due to the 1% excitations of Chi-Chi earthquake which demonstrates the results of the simulation which are in excellent agreement with those given analytical method and experimental data.

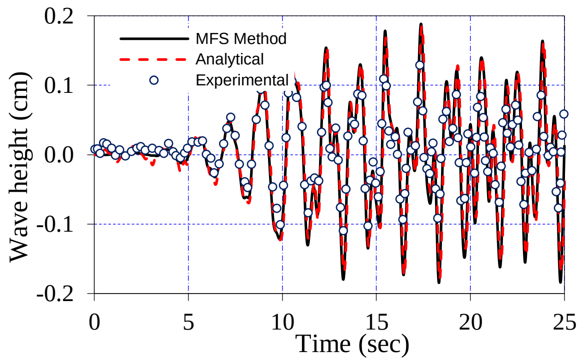

Figure 1. Wave elevations on right lateral wall of the rectangular tank subjected to seismic excitation of 1% Chi-Chi earthquake.

Conclusions

This paper presents a novel meshless MFS procedure based on Lagrangian form of pressure equation to simulate the liquid motion within a rectangular tank under harmonic and seismic loading. Lagrangian method is capable to precise geometry updating due to earthquake excitations and also can reduce solution errors by interpolating boundary points. To validate the proposed meshless numerical scheme, a rectangular water tank is investigated and compared with numerical experiments and boundary discretization numerical method such as BEM method. Solution is well compared with the results and showed the water sloshing and base shear force in a small water rectangular tank. The boundary conditions are imposed through a collocation approach and thus the method can be categorized in meshless types.

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