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New Kelvin-Voigt Model to simulate the collision of rigid bodies

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ABSTRACT: The effective simulation of collision between two adjacent structures has always been of interest in structural engineering. Several analytical models have been proposed by different scholars to simulate this phenomenon. The Kelvin-Voigt Model is one of the popular ones due to its linearity, and ease of application. In this study, a parameter (coefficient α) is introduced in the original Kelvin-Voigt Model to calculate the energy dissipation both in the compression and restitution (separation) phases of the contact. Besides, the accuracy of the modified model has been improved by presenting a new equation for the estimation of damped energy. Furthermore, the tensile force is eliminated in the restitution phase. The effectiveness of the proposed modified model in the simulation of the collision was examined by comparing the results with those of the original model, as well as previous experimental studies. The mean relative error between the selected coefficient of restitution before the collision and the coefficient of restitution after the collision that was evaluated by different models were compared. The modified model proposed in this study showed the least error values among all of the other models. This indicated the ability of the model to estimate the damped energy with better accuracy. The results of this research study indicate that the proposed modified Kelvin-Voigt Model is effective in the simulation of collision between two structures.

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1- Introduction

Collision is a physical phenomenon in which two or more objects come into sudden contact with one another. As a result of a collision, significant forces are exchanged between the colliding objects, and this may cause substantial stress demands and damages. The evaluation of stresses and strains due to collision is essential in many practical applications, including civil and mechanical engineering. The proper simulation of collision is an important step toward the identification of its effects on the colliding structures and providing a suitable solution to mitigate the consequences of impact in the structures. Various practical models have been proposed to simulate the impact of structures. Most of these models are inspired by the basic Hertz Model [1]. One of the most popular models in evaluating the impact force is the Kelvin-Voigt linear viscoelastic model.

In this paper, the linear viscoelastic Kelvin-Voigt Model is introduced and its pros and cons are identified. Next, modifications have been applied to the model to improve its accuracy in estimating the energy dissipations during the collision as well as simulating the time history of the impact force.

2- Methodology

Hertz's basic theory [1] expresses the force of a collision between two objects, F_p, as a nonlinear power function of the relative deformation of the colliding objects.

$$F_p(t) = k\,\delta(t)^n \tag{1}$$

In Equation (1), k is a stiffness parameter and δ(t) is the relative deformation of the two colliding objects. The model accounts for elastic deformations only and neglects energy dissipation. If the power n in Equation (1) is set to unity, a linear elastic model is created. The linear viscoelastic model (Kelvin-Voigt Model) is constructed by modifying the linear elastic model of Equation (1). As such, in addition to the linear spring, a linear dashpot is also used in parallel to the spring to simulate the energy dissipations (Equation (2)) [2]. In Equation (2), c(t) represents the viscous damping coefficient, which is calculated from Equation (3). Parameter $\dot{\delta}(t)$ indicates the relative velocity of the two colliding objects.

$$F_p(t) = k\,\delta(t) + c(t)\,\dot{\delta}(t) \tag{2}$$

$$c(t) = 2\zeta\sqrt{k\frac{m_1 m_2}{m_1 + m_2}} \tag{3}$$

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Table 1. Comparison of different variants of Kelvin-Voigt Model in estimating the coefficient of restitution e

Models	$Ave.Error = Ave.\left(\frac{\lvert e_{PRE} - e_{POST} \rvert}{e_{PRE}}\right) \times 100$	
	$0.1 \leq e \leq 1$	$0.5 \leq e \leq 0.8$
S. Anagnostopoulos [3]	28.76	11.65
S. Mahmoud & R. Jankowski [4]	9.41	9.04
K. Ye et al. [5]	10.86	4.60
Current paper	3.68	0.68

Table 2. Comparison of the accuracy of different variants of Kelvin-Voigt Model in the peak response evaluation of the impact force

Models	Absolute value of error (%)
S. Anagnostopoulos [3]	2.2
S. Mahmoud & R. Jankowski [4]	3.8
K. Ye et al. [5]	0.8
Current paper	0.8

Parameter ξ in Equation (3) represents the equivalent viscous damping ratio. One of the most commonly employed relations to evaluate ξ is provided by Anagnostopoulos [3] as follows.

$$\zeta = \frac{-\ln e}{\sqrt{\pi^2 + (\ln e)^2}} \tag{4}$$

In the model presented by Mahmoud and Jankowski, the damping ratio is calculated as follows [4].

$$\zeta = \frac{1 - e^2}{e(e(\pi - 2) + 2)} \tag{5}$$

Using Hertz theory and the first principles of structural dynamics, Ye [5] presented a model for modifying the Kelvin Model in which the damping ratio is considered as follows.

$$\hat{\zeta} = \frac{3}{2} \frac{k(1-e)}{e(\dot{\delta}_i)} \tag{6}$$

The energy dissipation in the impact (approach) phase is greater than the separation (restitution) phase [6]. According to the classical theory of collision and the law of conservation of energy and momentum, the damping ratio in this paper is presented in terms of α as follows:

$$\zeta = \frac{3}{2} \frac{(1-e^2)}{(\alpha + e^2)} \frac{k_h}{\dot{\delta}_i} \tag{7}$$

The parameter α varies between 0 and 1. The value of 0 for α indicates that the energy dissipation is not included in the separation phase, and the value of 1 means that the energy dissipation of the compression phase and the separation phase is considered to be the same. Parameter α should be ideally selected such that the initial value of coefficient e, which is usually evaluated empirically for different substance materials, and the value of e after the collision, remain unchanged. As such, by determining an appropriate value for parameter α in the modified Kelvin-Voigt Model of this paper, the effect of energy dissipation in both collision and separation phases is taken into account.

The final value of coefficient α is evaluated using an iterative procedure as follows. First, the initial value of e (i.e., the pre-collision value, e_{pre}) is determined from the literature or via experiment based on the substance material of the colliding object. It should be noted that the value of the return coefficient e is unique for each substance material. Next, the impact force is evaluated based on the Kelvin-Voigt Model. The magnitude of the impact force is affected by the value of the unknown coefficient α which is varied between 0 and 1. The values of the coefficient α and the effective impact force are evaluated through an iterative procedure that aims at minimizing the error between the post-collision e (i.e., e_{post}) and pre-collision e (i.e., e_{pre}) values. The return-coefficient of various substance materials that are mostly employed in different practical engineering applications is located within the approximate range of 0.5 to 0.75 [7]. The coefficient α for the case study of a concrete ball dropping on a concrete slab can be expressed as a function of e as follows.

$$\alpha = f(e) : \begin{cases} e+1 & 0.1 \leq e < 0.8 \\ 1 & 0.8 \leq e \leq 1 \end{cases} \tag{8}$$

The model of a concrete ball falling on a concrete slab as presented in this paper was aimed at demonstrating the iterative procedure that can be employed to evaluate the coefficient α. The coefficient α acts as an adjusting valve that balances the energy dissipations between the collision and separation phases of impact. It is noteworthy that in the present study, it is assumed that the materials remain linear elastic during and after the collision. Therefore, any plastic behavior or stiffness degradations following the impact is neglected. In addition, the effects of successive impacts are not accounted for in the analysis.

3- Results and Discussion

Table 1 compares the accuracy of different variants of Kelvin-Voigt Models, including the model presented in this paper, in evaluating the pre- and post-collision return-coefficients and the peak impact force in the problem of the falling of a ball on a slab of the same substance material for various range of e-values. As seen, the model presented in this paper is more accurate. In Table 2 the accuracy of different models in the evaluation of the peak impact force of a concrete ball that is fallen on a concrete slab is presented. As seen, the model presented in this paper is in better agreement with the experimental results.

4- Conclusions

The accurate simulation of collision is a critical step to evaluate the stress demands and mitigate the response. In this research, a new variant of Kelvin-Voigt Model capable of balancing the energy dissipations before and after the collision was presented. The model employs an α-coefficient to adjust the energy dissipation during the collision and separation phases of the impact. The results of this study suggest that the proposed model is effective in the time history simulation of the impact force of colliding objects.

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