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Evaluation of Non-sway Flexural Buckling of One-bay Gabled Frames by Solving Characteristic Equation

A. Haghollahi*[1], A. A. Safavi[2]

[1] Civil Engineering Department of Shahid Rajayee Teacher Training University, Tehran, Iran.
[2] Lecturer of Engineering Department of Golestan University, Fazel-Abad, Ali-Abad Katool, Iran

ABSTRACT: Flexural buckling is one of the buckling limit states in columns, which have at least one symmetric axis. Due to the lack of analytical solution for the differential equation of deformation of a non-prismatic column, its flexural buckling load has been determined by numerical methods, resulting in approximate solutions. This research aims at the analytical evaluation of non-sway in-plane flexural buckling of gabled frames. The equilibrium and differential equations were simultaneously used in the elastic flexural energy, consequently the characteristic equation is achieved. The effective length coefficient can be determined only with having two geometrical parameters of a gabled frame, using the relevant graph. Accurate results and simple use of the drawn graphs are among the benefits of the introduced method.

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1- Introduction

Stability analysis of tapered columns is difficult and researchers often use numerical and approximate methods to solve this problem. The old documented solutions for determining the critical load of tapered columns go back to early 19th century by Timoshenko [1], Morley [2] and Dinnik [3]. In all of these solutions, the tapered columns were approximated by stepped columns resulting in approximate solutions. The critical load calculation of tapered columns by Bessel's functions was first studied by Gere and Carter [4]. Iremonger solved the differential equation of deformation of a tapered column for arbitrary boundary conditions by finite difference method [5]. Karabalis and Beskos used Finite elements to obtain numerical solutions [6]. Comprehensive review of stability of the tapered columns has been separately done by Ermopoulos and Banerjee [7, 8]. Williams and Aston [9] studied several columns with variable web and flanges under concentrated loads. Non-prismatic columns were analyzed for buckling based on vibration modes, energy approaches and the Principle of Stationary Potential Energy by Rahai and Kazemi [10]. Bradford and Valipour [11] introduced shape functions for beams with elastic bases using the Principle of Virtual Work. Assuming the column displacement function as an exponential function of the summation of a power series, Darbandi et al [12] studied the tapered columns buckling by a perturbation method. Buckling and vibration of tapered members were studied by Taha and Essan using differential quadrature method [13]. In this method, the values of the first and higher derivatives of the answer function in a point are assumed to be equal to the summation of the weighted values of that function in other sample points. Ruocco et al. developed the Hencky bar-chain model for the buckling analysis of non-uniform columns [14]. In this model, a column is divided into some rigid columns; each two adjacent rigid columns are connected into each other with a rotational spring. The buckling load is determined by the equality of determinant of the stiffness matrix to zero. Nikolic´ and Šalinic performed the buckling analysis of columns with continuously varying cross-sections [15]. They used a rigid multi-body approach, which was similar to Hencky bar-chain model, but there were rotational and lateral translational springs between each two adjacent rigid columns.

2- Methodology

The structures shown in Figs. 1 and 2 have a concentrated force (P) applied vertically downwards to the top of each column. By assuming linear-elastic behavior in bending for a segment of the two buckled columns in Fig. 3, their lateral displacement functions, v, can be expressed by the following

*Corresponding author's email: haghollahi@sru.ac.ir

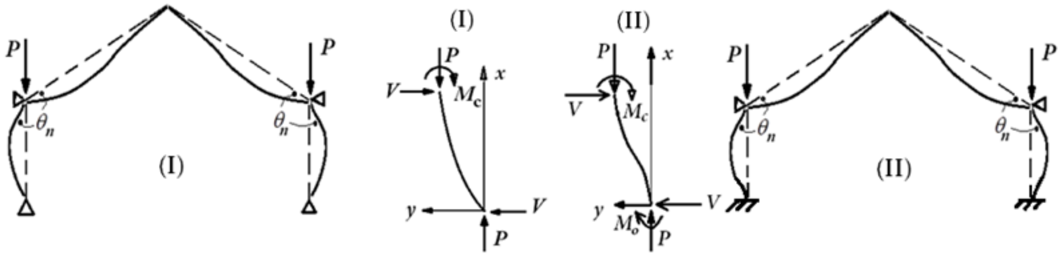

Fig. 1. The loadings, buckling mode configuration and the free body diagrams of studied gabled frames (Frame (I&II))

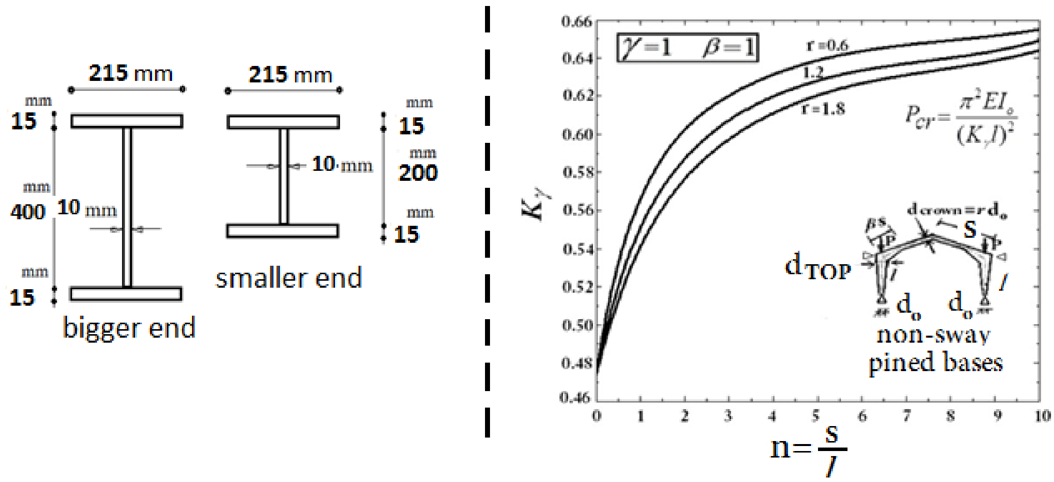

Fig. 2. A pined-pined tapered column with its end sections (left)- relevant graph (right)

differential equations (the equations numbers of frame (II) are indicated with "prime" indication):

$$M_c = Pv - Vx \qquad = -EI_c v'' \tag{1}$$

$$M_c = Pv - Vx - M_0 = -EI_c v'' \tag{1'}$$

Where E and I_c are the elasticity modulus of the members and the inertia moment of the bending axis of the column, respectively. Other parameters are shown in Fig.1. About the external work in the buckling and its equality with the summation of the elastic energy the next equation can be written:

$$W_{ext} = 2\times(P\times\Delta_h) \approx 2\times P\left(\frac{1}{2}\int_0^l v'^2\right) = U_c + U_b \tag{2}$$

Where Δ_h and U_c are the displacement of the column tip, the total elastic energy of the columns and the beam, respectively (U_b is the total elastic energy of the beams). The total elastic energy of the columns can be written as:

$$U_c = \int_0^l \frac{M_c\times M_c}{EI_c}dx = \int_0^l \frac{(Pv - Vx)\times(-EI_c v'')}{EI_c}dx \tag{3}$$

$$U_c = \int_0^l \frac{M_c\times M_c}{EI_c}dx = \int_0^l \frac{(Pv - Vx - M_0)\times(-EI_c v'')}{EI_c}dx \tag{3'}$$

By integration by parts and considering the boundary values, U_c can be determined:

$$U_c = \int_0^l -(Pv - Vx)v''dx$$

$$\Rightarrow U_c = P\int_0^l v'^2\,dx - Vl\theta \tag{4}$$

$$U_c = \int_0^l -(Pv - Vx - M_0)v''dx$$

$$\Rightarrow U_c = P\int_0^l v'^2\,dx - \theta(Vl + M_0) \tag{4'}$$

On the other hand $W_{ext} = P\int_0^l v'^2 dx$ and knowing $W_{ext} = U_c + U_b$, it can be resulted that $U_b = V\theta$ and for structures (I) and (II), $U_b = (M_0 + V)\theta$ respectively. Then by some mathematical calculations next two equations can be written:

$$U_b = \int_0^s \frac{[Vl + (V-F)x\sin\alpha]^2}{EI_b}dx = Vl\theta \quad (5)$$

$$U_b = \int_0^s \frac{[M_0 + Vl + (V-F)x\sin\alpha]^2}{EI_b}dx = (M_0 + Vl)\theta \quad (5')$$

After solving Eqs. (1) and (2) it can be realized that the rotation of the tip of the columns (θ) and M_o (base bending moment in Fig. 1) is dividable by "V", besides . $\theta = -v'_l$ Therefore:

$$\text{Structure (I):}\ f(P) = \frac{v'_l}{V} = -\frac{\theta}{V} \quad (6)$$

$$\text{Structure (II):}\ g_1(P) = \frac{v'_l}{V} = -\frac{\theta}{V}, g_2(P) = \frac{M_0}{V} \quad (6')$$

The final equations are the characteristic equations:

$$\text{Structure (I):}\ f(P) + \frac{l}{E}(I_3 - \frac{I_4^2}{I_5}) = 0, \quad (7)$$

$$\text{Structure (II):}\ g_1(P) + \frac{[g_2(P) + l]}{E}(I_3 - \frac{I_4^2}{I_5}) = 0 \quad (7')$$

Where $I_3 = \int_0^s \frac{dx}{I_b}, I_4 = \int_0^s \frac{x}{I_b}dx, I_3 = \int_0^s \frac{x^2}{I_b}dx.$

3- Results and Discussion

Having the characteristic equations drawing the required graphs will be possible. The graphs are drawn for effective length coefficient. By the equation $P_{cr} = \pi^2 E_0 (K_\gamma l)^2$, and solving the characteristic equations, intended graphs could be drawn.

3.1-Example

The effective length coefficient of a pined-pined column is requested. Its two end sections are shown in Figure 1.

3.2-Solution

By using the graph of Figure 1 can easily solve the problem. The hinged base at the bigger end (column top section) is similar to a very long oblique beam ($s \rightarrow \infty$) in Figure 1. By extrapolating the top curve: $K_\gamma = 0.6$

4- Conclusion

In this article the effective length coefficient of tapered columns in gabled frames is calculated. The introduced method is analytic and the use of the drawn graphs is very simple.

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