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Comparison of Convergence Speed among Conjugate Directions Traffic Assignment Algorithms

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ABSTRACT: The Traffic assignment problem in the transport networks is formulated as a convex optimization problem using simplifier assumptions. Link-based, path-based and bush-based algorithms have been presented to solve this problem, among which the link-based algorithm has found more applications thanks to its low memory requirements. The link-based algorithm of Frank-Wolf (FW) is yet amongst the most popular assignment algorithms, because of its simplicity as well as high convergence speed during the initial iterations. However, the low convergence rate of this algorithm near the optimal solution has driven many studies that focus on modifying the FW search direction, resulted in newer link-based algorithms. Among them are conjugate direction algorithms which are more effective and simply implementable. These algorithms include PARTAN, conjugate FW (CFW) and bi-conjugate FW (BFW). This paper uses the Chicago Regional and Sioux-Falls test networks to make direct comparisons among these algorithms with respect to the CPU time and iteration number needed to reach various accurate solutions. The results for Chicago show that, compared with the FW algorithm, the three algorithms CFW, PARTAN increase the convergence speed to the relative error of 10-5 (i.e. a stable solution) by about 89, 72 and 63%, respectively, while only the BFW can reach to a 10-6 relative error. Comparing the results from Sioux-Fall with those of Chicago demonstrates that the performance of the conjugate directions Algorithms improves as network size decreases.

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1- Introduction

The issues existing in human and freight transportation are among the most important and fundamental problems of the present age. Moreover, these issues are extensively increasing due to the increase in the value of time as well as the formation of competitive markets in this area. Therefore, it is necessary to conduct continuous studies with the aim of improving or modifying existing transportation systems. The main part of such studies is modeling the traffic of transportation networks in different situations of travel demand and transportation systems.

Link flow patterns in a transportation network are obtained by solving a mathematical program called the traffic assignment problem (TAP). Given the amount of travel demands between the origin-destination (OD) pairs in the network, the TAP is aimed to answer how demands are distributed among existing paths connecting the OD pairs. This problem fundamentally obeys the used equilibrium principle [1] that is no user can reduce his own travel time by unilaterally changing his path.

Knowing that the evaluation of transportation policies (such as network expansion and congestion pricing) requires so many times solving the TAP, providing fast convergent TAP algorithms has always been the focus of researchers in this field. Various solution methods have been proposed, differing more or less in fundamentals. They are generally divided into three categories: link-based, path-based and bush-based algorithms. The most famous link-based algorithm is the Frank-Wolf (FW) algorithm [2], which has been the most widely used TAP algorithm for the last five decades. The most important feature of this algorithm is its simplicity and low memory usage. Despite these positive features, the convergence rate of the FW algorithm is very low, especially when approaching the optimal solution.

2- Methodology

In this study, we investigate the convergence rate of the most important link-based TAP algorithms, including FW, PARTAN, conjugate FW (CFW) and Bi-conjugate FW (BFW) algorithms. The details of this algorithm can be found in [3].

Obviously, algorithms can be evaluated from different aspects, but without a doubt the convergence rate is one of the most important indicators in this type of evaluation. In this paper, to determine the convergence rate of the algorithms,

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Table 1. Specifications of the test networks

Network	No. of Zones	No. of Nodes	No. of Links	Total Demand
Sioux Falls	24	24	76	360.6
Chicago	1790	12,982	39,018	1,360,428

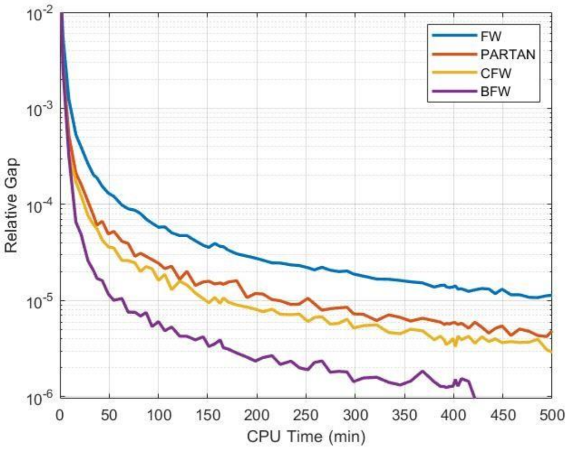

Fig. 1. Convergence rate of FW, PARTAN, CFW and BFW algorithms for Chicago network

the number of iterations and the time required to achieve a certain precision in solving the problem are used. In order to make correct comparisons between algorithms, a common structure must be used in their programming so that their results are comparable. Therefore, the computer codes of the aforementioned algorithms are written in a common structure with the same programming language, so that there is no difference between them except in the method of determining the search direction. In coding these algorithms, the bisection method [4] is used to perform linear search and the Golden algorithm [5] is used to solve the shortest path problem.

The algorithms are tested on two different networks, each of which has been used for a specific purpose. The small-sized Sioux Falls network is used to examine the solutions of these algorithms against each other as well as to control coding accuracy. Finally, the Chicago network is used as a large-scale network to compare the convergence rates of the competing algorithms. The characteristics of the test networks including the number of zones, nodes, links and sum of the elements of the demand matrix, are shown in Table 1 [6-8].

The intended algorithms were implemented in C++ programming environment by Visual Studio 2013 software on a computer with a 2.8GHz dual-core processor and 8GB of RAM. The relative gap (RGap) measure is used to set the convergence criterion, the most common criterion used in recent TAP algorithms. This measure is equal to the relative difference between the total travel time of the users and the total travel time of them as if they experienced the shortest path travel time on the network. According to a study by Boyce et al. [9], link flows in large-scale networks stabilize after an RGap of 10^{-5} is reached. Therefore, in this study, for both the Sioux Falls and Chicago networks, the RG criterion is defined as reaching an RG of 10^{-5} or less in order to obtain adequate accurate solutions.

Table 2. Results of FW, PARTAN, CFW and BFW algorithms to the precision of 10-5 for Chicago network

Algorithm	No. of iterations	CPU Time (min)	RGap
FW	6050	504.47	0.00000971
PARTAN	2249	187.85	0.00000984
CFW	1659	140.24	0.00000978
BFW	642	53.27	0.00000950

Table 3. Required memory of the studied algorithms for Chicago network (MB)

FW	PARTAN	CFW	BFW
0.312	0.624	0.468	0.624

Table 4 Relative number of iterations of the studied algorithms to the precision of 10-5 for Chicago and Sioux Falls networks

Network	FW	PARTAN	CFW	BFW
Chicago	1	0.37	0.27	0.11
Sioux Falls	1	0.35	0.18	0.02

3- Results and Discussion

Figure 1 shows the convergence rates of the FW, PARTAN, CFW, and BFW algorithms with respect to the CPU time until RGap of 10^{-5} is achieved. As can be seen, only the BFW algorithm is able to hit the RGap of 10^{-6} in a reusable time. Moreover, Table 2 tabulates the number of iterations as well as the CPU, and Table 3 reports the memory required by each algorithm to reach this precision.

Table 4 shows the results of a direct comparison between competing algorithms in terms of the number of iterations up to the RGap of 10^{-5} for the Chicago and Sioux Falls networks. In this table, the number of iterations of the PARTAN, CFW and BFW algorithms are normalized with respect to that of the FW algorithm.

4- Conclusions

In this paper, the speed of convergence of FW, PARTAN, CFW, and BFW algorithms to stable solutions were investigated. Comparisons among the algorithm were performed under the same software and hardware conditions on the large-scale Chicago network. The results showed that the three algorithms BFW, CFW and PARTAN can reach relative gaps less than 10^{-5} by about 89, 72 and 63%, respectively, of the CPU time the FW algorithm needed. In other words, conjugating the search directions (even approximately) increased the efficiency of the base FW algorithm by more than 60%. The efficiency of the BFW algorithm was much higher among the studied algorithms, due to the fact that it applies the conjugation with respect to the last two directions. The CFW algorithm converges slower than the BFW algorithm, because in which the conjugation is performed only relative to the previous direction. In terms of performance, the PARTAN algorithm was placed between the CFW and FW algorithms, because it approximates the conjugation and also requires an additional linear search per iteration. By the way, among the three conjugate direction algorithms, only the BFW was able to find a solution with a relative gap of less than 10^{-6} in less than 500 minutes.

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