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# The Effect of Linearization of Hoek-Brown Criterion on the Bearing Capacity of Rock Masses using the Upper Bound Method of Limit Analysis

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**ABSTRACT:** One of the most important issues in the calculation of the bearing capacity of rock masses is the method of application of the rock mass failure criterion. The Hoek-Brown failure criterion is the most useful criterion in practical applications. For applying this criterion in the upper bound method of limit analysis, one should linearize it using the single or multi-tangential technique. In this paper, the method of linearization of the Hoek-Brown criterion is investigated to determine the bearing capacity of embedded footings on rock masses. Since different stress levels have existed in the rock mass body, the multi-tangential technique results in the best approximation of the nonlinear Hoek-Brown criterion. As a novelty for the current research, the embedment depth of the footing is considered directly in the upper bound formulations instead of replacing it with an equivalent surcharge. The obtained results show that considering the embedment depth of footings along with using the multi-tangential technique result in increasing the accuracy of the results. In the methods which consider the embedment depth as an equivalent surcharge, the extension of the failure lines through the rock mass above the footing base cannot be considered.

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## 1- Introduction

The design of footings on rock masses involves investigating various aspects, one of which is the determination of bearing capacity. In this paper, the effect of linearization of Hoek-Brown criterion on the bearing capacity of rock masses was investigated using the upper bound method of limit analysis. The effect of footings embedment depth was also considered in the analyses. Finally, comprehensive sensitivity analyses were performed to determine the effects of different parameters on the bearing capacity of rock masses.

## 2- Methodology

The failure mechanism considered in the present paper is shown in Figure 1. Due to the symmetry, only the central wedge and half of the failure mechanism are shown. This mechanism has the capability of considering the embedment of the footing. The number of wedges in the mechanism was obtained during the optimization process in order to result in the best (lowest) value of the ultimate bearing capacity.

In this paper, the modified Hoek-Brown nonlinear failure criterion was used which is the most practically applicable criterion for analyzing rock mass behavior and provides a good agreement with the experimental results. Despite its original nonlinear form, this criterion was also linearized in analyzing stability problems. Two common methods were used by different researchers for linearizing the Hoek-Brown criterion in the upper bound method which include the tangential line method [1-3] and the multi tangential technique [4-8]. In these two methods, the nonlinear Hoek-Brown criterion is replaced by one or several tangential lines, respectively. Each one of these tangential lines has a unique slope and y-axis intercept, which correspond to the internal friction angle and the cohesion of the rock mass, respectively. Obviously, in the tangential line method, a constant value of the internal friction angle and the cohesion is obtained for the whole rock mass, whereas in the multi-tangential technique, different values of friction angle and cohesion are obtained for the rock mass according to the level of stress.

In order to calculate the bearing capacity by the upper bound method, the total external work performed in the mechanism should be equated to the internal energy dissipated through the velocity discontinuity lines. By doing so, the equation of the ultimate bearing capacity of rock masses was obtained as follows:

$$q_u^D = s^{0.5}\sigma_{ci} N_\sigma^D + qN_q^D + \frac{1}{2}\gamma B N_\gamma^D \tag{1}$$

Where $s$ is the Hoek-Brown parameter which depends

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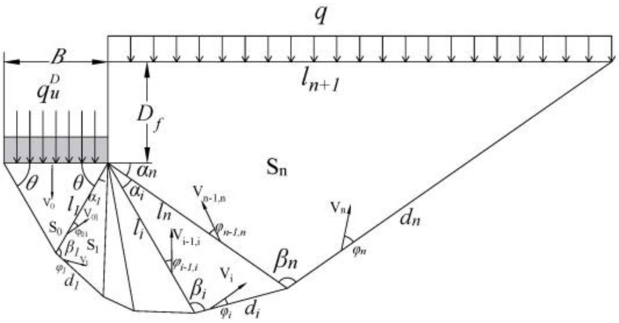

**Fig. 1. Multi-block failure mechanism and the velocity vectors**

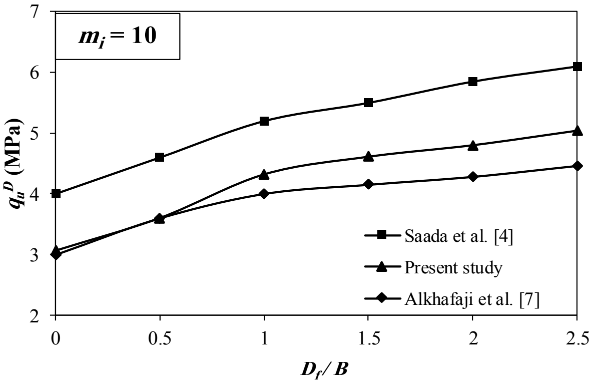

**Fig. 2. Comparison of the bearing capacities obtained from the methods which are based on the multi-tangential technique**

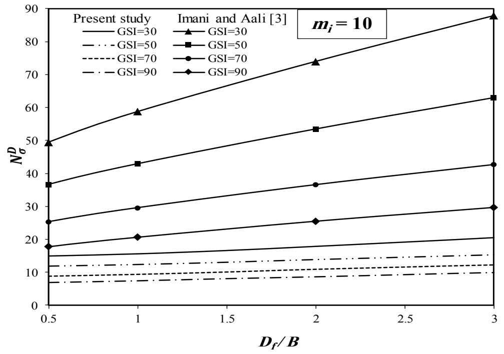

**Fig. 3. Comparison of the bearing capacity factors obtained from the multi-tangential and tangential methods**

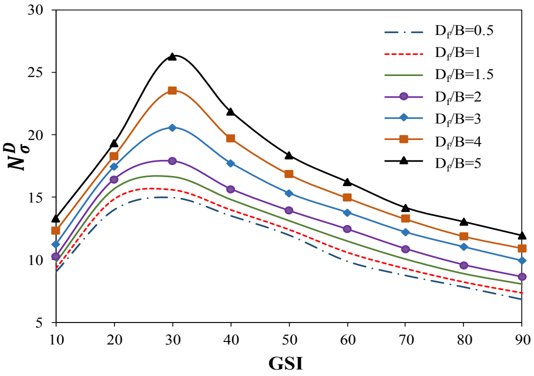

**Fig. 4. Variation of $N_\sigma^D$ versus GSI assuming $m_i = 10$**

on GSI (Geological Strength Index), $\sigma_{ci}$ is the uniaxial compressive strength of the intact rock, $q$ is the surcharge, $\gamma$ is the unit weight of the rock mass, $B$ is the footing width and $N_\sigma^D$, $N_q^D$, $N_\gamma^D$, are the bearing capacity factors. Since the uniaxial compressive strength of the intact rocks is commonly high, the effect of surcharge and unit weight of the rock masses do not have a considerable effect on the bearing capacity. Therefore, Eq. (1), changes to the following form:

$$q_u^D = s^{0.5}\sigma_{ci} N_\sigma^D \tag{2}$$

Therefore, the bearing capacity factor, $N_\sigma^D$, can be written as follows:

$$N_\sigma^D = \frac{q_u^D}{s^{0.5}\sigma_{ci}} \tag{3}$$

## 3- Results and Discussion

The upper bound formula of the bearing capacity should be optimized to achieve the best (lowest) magnitude of the bearing capacity. Using the optimization process, the optimum number of wedges was obtained equal to 19. The results of this study were compared with the results of other researchers who have used the multi-tangential technique. Assuming $\gamma = 21$ kN/m³, $D = 0$, and GSI = 30, Figure 2 shows that the bearing capacity obtained from the present study is more than that of AlKhafaji et al. [7] method. The maximum difference between these two methods is equal to 14%. Also, the bearing capacity obtained from the present paper is lower than that proposed by Saada et al. [4], with the maximum difference equal to 22 %.

Figure 3 compares the results of the present paper with that of Imani and Aali [3] which is based on the tangential linearization of the Hoek-Brown criterion. It was assumed that the Hoek-Brown constant, $m_i$, is equal to 10. It is clear that using the multi-tangential technique in the present research results in smaller values of $N_\sigma^D$ and the corresponding bearing capacity which is of paramount importance in practical applications.

Assuming $\sigma_{ci}$ = 10 MPa and $D$ = 0, Fig. 4 illustrates the variation of $N_\sigma^D$ versus GSI. As can be seen, the $N_\sigma^D$

increases with increasing the GSI from 10 to 30, and decreases from 30 to 90. The same trend can be seen in the previous studies [1-3, 8, 9]. For a constant value of GSI, the $N_\sigma^D$ are larger for higher $D_f/B$ ratios.

## 4- Conclusion

In this paper, the effect of linearization of Hoek-Brown criterion on the bearing capacity of rock masses was investigated using the upper bound method of limit analysis. A formula was proposed for the bearing capacity of rock masses considering the multi-tangential technique for linearizing the Hoek-Brown criterion. The obtained results show that linearizing the nonlinear Hoek-Brown criterion with a single line, i.e., the tangential method, results in unreliable bearing capacity. However, the multi-tangential technique can considerably improve the bearing capacity of rock masses. Among different parameters affecting the bearing capacity factor, $N_\sigma^D$ , GSI, $m_i$, and $D$ have the highest influence.

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