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Formulating a new efficient simple element for statics, buckling and free vibration analysis of Timoshenko's beam

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ABSTRACT: The beams are really useful for a large number of engineering structures. In this article, a simple, robust beam element will be formulated. Other researchers utilized several theories such as Euler-Bernoulli, Timoshenko and higher-order shear for analyzing the beams. The proposed formulation will be written based on satisfying the equilibrium equation. Using the equilibrium equation reduces the number of unknowns in addition to improving the efficiency of the new element. The suggested element has only two nods and two degrees of freedom per node. The third and second-order polynomials will be used for vertical displacement and rotation fields, respectively. After calculating the matrix of shape functions, the governing equations of statics, free vibration and buckling analysis can be written. Finally, using the suggested element, static analysis, free vibration and buckling were performed on several problems. To prove the efficiency of the new element, a large number of benchmark tests will be utilized. These numerical tests have various support conditions and different aspect ratios. With the help of these tests, rapid convergence and high accuracy of the proposed element will be shown. The new element has high efficiency in all of the static, free vibration and buckling analysis for both thin and thick beams besides its simplicity. Good element answers of other researchers will be available to have a better comparison. .

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1- Introduction

The beams are common structural members in most engineering structures. The static and dynamic characteristics of beams are evaluated using classical deformation theories or modified shear deformation theories. The first theory of beam bending was based on the Euler-Bernoulli hypothesis. This theory overestimates the buckling load of the beam by ignoring the effects of shear deformation and the concentration of transverse stress. Hence, this theory applies only to narrow beams. In this approach, by increasing the thickness of the beam and shear effect deformation, the error of response is increasing. The next method used in bending beam analysis is the Timoshenko beam theory or the first-order shear deformation theory, which can take into account the shear deformation effect to some extent. In this theory, since it is difficult to calculate the actual transverse shear stress distribution on the cross-section to assume a plane cross-section after deformation, a shear correction factor is needed to correct the shear stiffness in the calculations. The effect of shear transformation is formulated in Timoshenko's theory. Therefore, this method has a better result, especially in deep beams in which the shear effect is impressive. Up to now, many elements have been presented based on Timoshenko's theory. These elements are classified into two groups which are simple and higher-order elements.

Li et al., by establishing a relationship between Timoshenko's beam theory and classical shear deformation theory, proposed a new strategy for determining the natural free vibrational frequencies of beams [1]. A new isogeometric method was developed based on Timoshenko's beam theory to study the free vibration of thick Timoshenko beam by Lee and Park. They used three modified methods in the beam analysis. They also identified shear locking errors in the analysis using numerical tests [2]. Arvin proposed a new relationship for the analysis of the free vibration of micro-rotating beams based on the strain gradient theory and the hypotheses of the Timoshenko and Euler-Bernoulli beam models. He used the differential transform method to obtain axial natural frequencies [3]. Isogeometric collocation methods for solving the problem of Timoshenko's beam were proposed by Veiga et al. Using this solution will eliminate the shear locking error[4]. Torabi et al. proposed a close-form solution for analyzing the free vibration of Timoshenko's beams under the arbitrary load [5]. A new Timoshenko's beam element was presented by Zhang et al. They analyzed the behavior of static bending, free vibration and buckling of Timoshenko's micro beams [6]. Hsu developed an enriched finite element method for the free-vibration analysis of various Timoshenko beam models. They used both generalized finite element (GFEM) and hierarchical finite element (HFEM) methods for element formulation[7].

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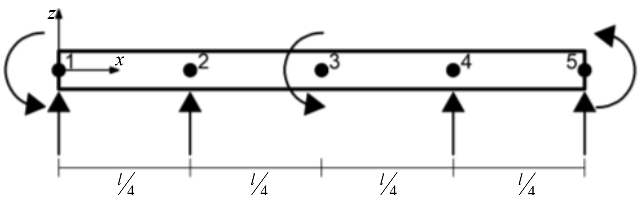

Fig. 2. Initial geometry of suggested element.

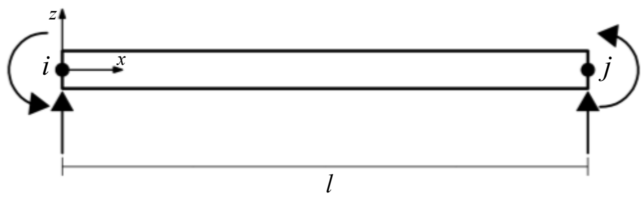

Fig. 3. Final geometry of proposed element.

In this article, a simple, robust beam element is formulated with the help of Timoshenko's theory. This formulation is written based on satisfying the equilibrium equation. Using the equilibrium equation reduces the number of unknowns in addition to improving the efficiency of the new element. The suggested element has only two nod and two degrees of freedom per node.

2- Proposed element's formulation

In the beginning, the third and second-order polynomials are used for vertical displacement and rotation fields, respectively. After calculating the shape functions' matrix, all of the fields were written based on the nodal unknowns' vector. The initial geometry of the proposed element is represented in Figure 1 . Equation (1) shows the governing relation for the Timoshenko beam. Using matrix form of suggested fields in equilibrium equation reduced the number of unknowns. A satisfying equilibrium equation creates a dependency between the nodal degrees of freedom as Equation (2). Where E, G, A, I, and l represent elasticity modules, shear modules, area, the moment of inertia, shear correction factor and length, respectively. By eliminating three degrees of freedom(, ,), the final geometry of the element has only two nodes and two degrees of freedom per node, same as Figure 2.

$$\frac{dw}{dx}=\phi-\frac{1}{k_sGA}\frac{d}{dx}\left(EI\frac{d\phi}{dx}\right)l \tag{1}$$

$$\left[w_2 \quad w_4 \quad \phi_3\right]^T=\mathbf{C}_{24}^{-1}\mathbf{C}_{13}\left[\phi_1 \quad \phi_5 \quad w_1 \quad w_5\right]^T \tag{2}$$

$$\mathbf{C}_{24}=\begin{bmatrix} \frac{9}{l} & -\frac{9}{2l} & -\frac{8EI}{k_sGAl^2} \\ -\frac{45}{l^2} & \frac{36}{l^2} & -\frac{4}{l} \\ \frac{81}{2l^3} & -\frac{81}{2l^3} & \frac{4}{l^2} \end{bmatrix} \tag{3}$$

$$\mathbf{C}_{13}=\begin{bmatrix} 1-\frac{4EI}{k_sGAl^2} & -\frac{4EI}{k_sGAl^2} & \frac{11}{2l} & -\frac{1}{l} \\ -\frac{3}{l} & -\frac{1}{l} & -\frac{18}{l^2} & \frac{9}{l^2} \\ \frac{2}{l^2} & \frac{2}{l^2} & \frac{27}{2l^3} & -\frac{27}{2l^3} \end{bmatrix} \tag{4}$$

With the help of these new fields, the strain energy function is calculated. By minimizing strain energy, the stiffness matrix will be available the same as Equation (5). Also, by utilizing Hamilton's principle and external work of axial force, the mass matrix and geometric stiffness matrix are obtained, respectively. These matrices are as below:

$$\mathbf{K}=\int_0^l\left[\begin{array}{l} EI\left(\frac{\partial \mathbf{N}_\phi}{\partial x}\right)^T\left(\frac{\partial \mathbf{N}_\phi}{\partial x}\right)+ \\ k_sGA\left(\begin{array}{l}\left(\frac{\partial \mathbf{N}_w}{\partial x}\right)^T\left(\frac{\partial \mathbf{N}_w}{\partial x}\right)-\left(\frac{\partial \mathbf{N}_w}{\partial x}\right)^T\mathbf{N}_\phi \\ -\mathbf{N}_\phi^T\left(\frac{\partial \mathbf{N}_w}{\partial x}\right)+\mathbf{N}_\phi^T\mathbf{N}_\phi\end{array}\right)\end{array}\right]dx \tag{5}$$

$$\mathbf{M}=\int_0^l\left[\left(\rho I\right)\mathbf{N}_\phi^T\mathbf{N}_\phi+\left(\rho A\right)\mathbf{N}_w^T\mathbf{N}_w\right]dx \tag{6}$$

$$\mathbf{K}_g=\int_0^l\left(\frac{\partial \mathbf{N}_w}{\partial x}\right)^T\left(\frac{\partial \mathbf{N}_w}{\partial x}\right)dx \tag{7}$$

Where $\mathbf{N}_w$ and $\mathbf{N}_\phi$ show shape function matrices of vertical deflection and rotation, respectively. Also, represents density.

3- Results and Discussion

For proving the excellent efficiency of the proposed element, several different benchmarks are used. In the beginning, calculating the frequency results of the new element compared with the isoparametric Timoshenko beam shows rapid convergence of the suggested element. The proposed element results in high accuracy even for coarse meshes. Then, the ability of the proposed element in free vibration analysis of beam with several support conditions is evaluated. For this purpose, six different support conditions containing clamped, pinned, sliding and free conditions were utilized. In addition, the beam in each support condition has been analyzed for seven different aspect ratios. Responses of good elements of other researchers in each one are available. Comparing the results of a new element with good elements of others shows the high accuracy of the proposed element even for higher modes' natural frequencies responses. For the exhibition the efficiency of new elements, static and buckling analysis is performed beside the free vibration numerical tests. Based on these benchmarks, the proposed element has an excellent performance also in the static and buckling problems.

4- Conclusions

In this article, a new beam element is proposed. Satisfying the equilibrium equation, in addition to the efficiency of the proposed element, reduces the number of unknowns. The simple suggested element has high accuracy and rapid convergence for all static, free vibration and buckling analyses. Several numerical tests with various boundary conditions and different aspect ratios for the beam are utilized to prove the efficiency of the proposed element.

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