



# A fuzzy approach for designing of subway lines, case study: development of the Tehran subway network

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**ABSTRACT:** Subway network design can be classified as one of the most challenging problems in transportation planning, where different deterministic or non-deterministic approaches have been utilized for optimal design. Non-deterministic methods, having fewer limitations and representing reality with its intrinsic uncertainty, have thus been the focus of less research. This paper incorporates concepts of fuzzy set theory into the optimal design of subway networks to the case of Tehran. Two binary mathematical programming models with different objective functions are developed. The first model maximizes the covered population while minimizing construction cost, whereas the second maximizes the ratio of the covered population to construction cost. These objective functions are modeled in both a fuzzy and a deterministic state. In the fuzzy model, we use a fuzzy penalty factor instead of edge length constraints and propose a Sugeno fuzzy inference system for calculating the covered population. Results indicate that the total length of designed lines with the linear and nonlinear fuzzy approach is equal to 139.3 km (477000 billion Iranian Rials) and 144.6 km (494000 billion Iranian Rials), respectively. Considering topology improvement per construction cost index, designed lines with the linear fuzzy model are better than the nonlinear fuzzy model. In comparison to the classic deterministic approach, the proposed fuzzy approach can improve topology improvement per construction cost index by 23 percent.

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## 1- Introduction

The Subway network is known as an essential and important transit mode in huge cities. Subway networks projects are major endeavors that require long-term planning. This paper proposes a fuzzy approach for designing subway lines.

## 2- Methodology

This paper compares four mathematical programming models, described in the following, for designing subway lines:

1. Linear mathematical programming (LP)
2. Non-linear mathematical programming (NLP)
3. Linear mathematical programming, including the proposed fuzzy penalty coefficient of edge length and the proposed fuzzy approach for population coverage calculating (FLP)
4. Non-linear mathematical programming, including the proposed fuzzy penalty coefficient of edge length and the proposed fuzzy approach for population coverage calculating (FNLP)

Formally, the metro network design problem is that of embedding a set of interconnected transit lines within a larger undirected network  $G=(N,E)$ , where  $N=\{1,..,n\}$  is a node-set

and  $E=\{(i,j): i,j \in N, i < j\}$  is an edge set. The nodes correspond to population centroids in a city, while the edges correspond to potential connections to be built between vertex pairs. The objective is a linear combination of the construction cost and of the population covered by the network. Let  $x_{ij}$  be a binary variable equal to 1 if and only if edge  $(i,j)$  belongs to the network, let  $y_i$  be a binary variable equal to 1 if and only if vertex  $i$  belongs to the network, and let  $a$  be a user-controlled positive parameter. Let  $T \subset N$  be a set of nodes that must necessarily belong to the network ( $T$  may be empty) and  $d_{ij}$  is a fuzzy penalty factor utilized instead of edge length constraint.

$$\min Z = \sum_i c_i y_i + \sum_{i,j} L_{ij} c_{ij} d_{ij} x_{ij} - \sum_i P_i y_i \quad (1)$$

$$\max Z = \frac{\sum_i P_i y_i}{\sum_i C_i y_i + \sum_{ij} L_{ij} c_{ij} d_{ij} x_{ij}} \quad (2)$$

$$\min Z = \sum_i c_i y_i + \sum_{i,j} L_{ij} c_{ij} x_{ij} - \sum_i P_i y_i \quad (3)$$

$$\max Z = \frac{\sum_i P_i y_i}{\sum_i C_i y_i + \sum_{ij} L_{ij} c_{ij} x_{ij}} \quad (4)$$

$$\sum_{i,j} x_{ij} = \sum_i y_i - 1 \quad (5)$$

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$$\sum_{i,j \in S} x_{ij} \leq \sum_{i \in S/\{k\}} y_i, k \in S \subseteq N, |S| \geq 2 \quad (6)$$

$$\sum_{i \in V/\{i\}} x_{ij} = y_i \quad (7)$$

$$\sum_{j \in V/\{j\}} x_{ij} = y_j \quad (8)$$

$$\sum_{i,j \in S} x_{ij} \geq y_k, k \in S \subseteq N, |S| \geq 2 \quad (9)$$

$$500 \leq L_{ij} \leq 2000; \forall ij \in E \quad (10)$$

$$y_i = 0, 1; i \in N \quad (11)$$

$$x_{ij} = 0, 1; i, j \in N \quad (12)$$

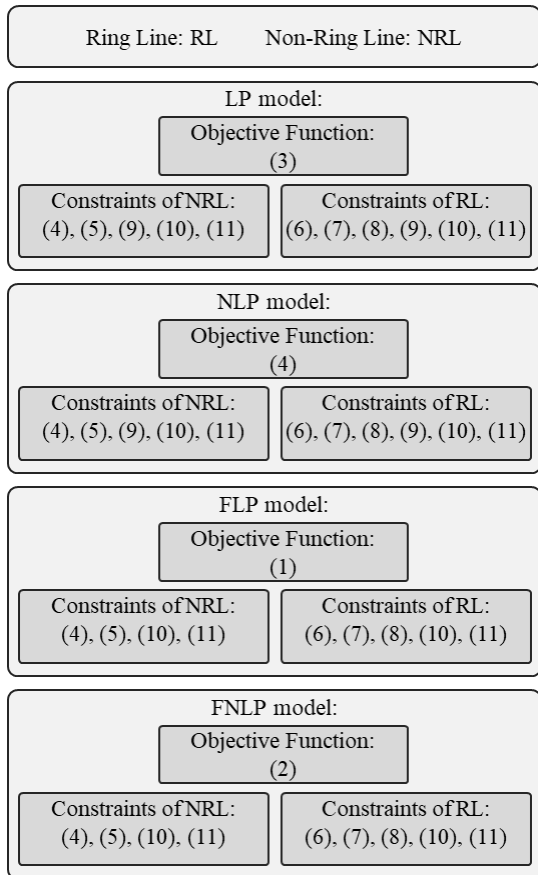


Fig. 1. Utilized mathematical programming models

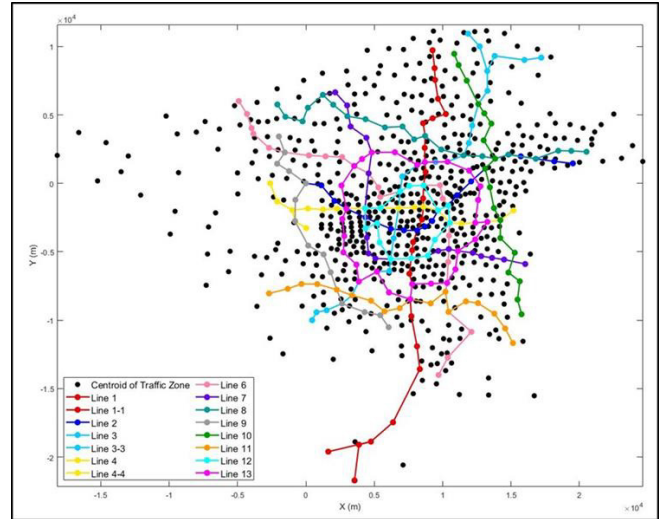


Fig. 2. Designed network based on FLP model

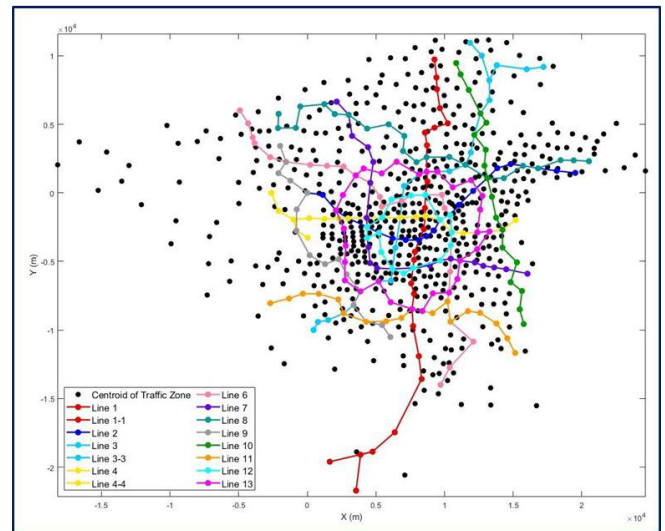


Fig. 3. Designed network based on FNLP model

Figure 1 depicts utilized models for designing subway lines. Designed networks evaluated based on assessment indices such as cycle availability, complexity, connectivity [1], population coverage, average edge length, station density (number of stations per square kilometer) and so on [2]. In this paper, the traffic zones data of Tehran is used for designing new ring or non-ring lines.

### 3- Results and Discussion

Currently, Tehran has seven subway lines and this paper proposes two new ring lines (around central regions of Tehran) and four new non-ring lines (around Tehran city).

Results of FLP and FNLP models indicate in Figures 1 and 2, respectively. The length of new lines designed based on FLP model is equal to 139.3 km (477000 billion Iranian Rials) while the length of new lines designed based on FNLP

**Table 1. Values of topology improvement per construction cost index**

<i>Model</i>	<i>TICC index</i>
<i>LP</i>	1.192
<i>NLP</i>	1.248
<i>FLP</i>	1.522
<i>FNLP</i>	1.476

model is equal to 144.6 km (494000 billion Iranian Rials)

Values of topology improvement per construction cost (TICC) index for utilized models are represented in table 1. Results indicate that FLP model has better performance than other models.

#### 4- Conclusions

In comparison to the classic deterministic approach, the proposed fuzzy approach can improve topology improvement per construction cost index by 23 percent.

#### References

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