Investigation the Stage-Discharge Relation and Discharge Coefficient in Sharp-Crested Weirs with Triangular Shape in Plan

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ABSTRACT

The triangular in plan sharp-crested weirs do not have a direct and straight edge and are in good agreement with the broken line. The aim of the present study is to provide an equation for discharge coefficient \(C_d\) for these types of weirs. \(C_d\) is between 0.53-0.88 based on the observed data. For \(\alpha=15\) degrees \(C_d\) has highest value and thus weir can convey maximum discharge. Using laboratory data based on \(h/p\) and \(\alpha\) parameters, a regression equation was presented. The results of the regression equation were compared with the results of the numerical model (Ansys Fluent) and the results showed the high precision of this equation. Ansys Fluent software works based on finite volume method. The numerical simulation is 3D. In addition, the performance of MR-Linear and MR-nonlinear regression models on the application of the stage-discharge equation the triangular in plan sharp-crested weirs were investigated and indicated that the result of this equations is very similar to results of the experimental data. The results also showed that due to the angle of the triangular in plan sharp-crested weirs, the \(C_d\) is increased from 1 to 8 % to the suppressed weir. In a situation where the head on the crest of these weirs is low, they will show better performance.

KEYWORDS

Triangular in plan sharp-crested weirs, Discharge coefficient, Stage-discharge, Ansys Fluent

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1. Introduction

There are many methods to measure the inflow of water into the channels and also to control the water level. The most common of these methods are weirs, flumes, orifices and measuring stations. Weirs are more commonly used because of their simple and relatively accurate relations. Weirs due to simplicity of construction and economically are one of the most common measurement structures as well as water level regulation in canals and rivers [1]. The first studies on the design and hydraulic performance of labyrinth weirs were performed by Hey and Taylor [2], and Taylor [3] on models with triangular, rectangular, and trapezoidal geometrical shapes with sharp edges. Distefano and Ferro [4] studied the overflow process through a triangular shape in the plan and obtained the stage-discharge relation for this type of structure using dimensional analysis and similarity theory. Norouzi et al. [5] studied a comparative study of artificial neural network and support vector machine to estimate the discharge coefficient of labyrinth weir. The performance of the MLP model with RMSE, R, and DC of 0.019, 0.985, and 0.971, respectively, was more acceptable and closer to the experimental data. In the present study, the performance of linear (MR-linear) and non-linear (MR-nonlinear) regression methods in determining the stage-discharge relation (correlation between \( h/P \) as a dimensionless quantity of stage and \( K_s/P \) as a dimensionless quantity of discharge) will be evaluated. Then, a numerical model will be developed using Ansys Fluent software to compare the results among the experimental results, the results of the proposed relation in the present study, and the results of other researchers.

2. Methodology

In order to determine the discharge in a rectangular sharp edge weir, Eq. (1) is established. Where \( Q \) is discharge, \( L \) is the total weir crest length, \( h \) is the total upstream head, \( g \) is the gravity acceleration and \( C_d \) is the discharge coefficient. By simplifying of Eq. (1), Eq. (4) is obtained.

\[
Q = \frac{2}{3} C_d \sqrt{2gh} L \ h^2
\]  

(1)

\[
h = \left(\frac{9}{8C_d^2}\right)^\frac{1}{2} \left(\frac{Q^3}{L^2g^2}\right)\]

(2)

\[
K_s = \left(\frac{Q^3}{L^2g^2}\right)\]

(3)

\[
h = \left(\frac{9}{8C_d^2}\right)^\frac{1}{2} \left(\frac{K_s^3}{P^2g^2}\right) \rightarrow C_d = \frac{3}{\sqrt{8}} \left(\frac{K_s}{P}\right)^\frac{1}{2}
\]

(4)

For a triangular sharp-edged overflow shown in Fig. (1), the stage-discharge relation can be expressed in the form of relation 5:

\[
\phi(h, Q, p, L, g, \mu, \rho, \sigma, B) = 0
\]

(5)

Figure 1. Schematic sharp-crested weirs with triangular shape in plan

Using Buckingham theory, Eq. (6) is derived:

\[
h = h\left(\frac{K_s}{P} \frac{L}{P} \frac{Re}{We} \frac{B^2}{L}\right)
\]

(6)

By simplifying the relationship 6 and \( B/L = \sin \alpha \), Eq. (7) is produced.

\[
h = h\left(\frac{K_s}{P} \frac{L}{P} \sin \alpha \right)
\]

(7)

Using the experimental results as observational data, a model similar to the laboratory conditions was developed. Numerical simulations were performed in the present study using the dimensions used in accordance with the experimental conditions in a 6 m long channel, 0.28 m wide and 0.41 m high.

Figure 3. Boundary conditions applied in the numerical model of the present study
3. Results and Discussion

In the present study, the relations of linear multiple regression and nonlinear multiple regression were extracted using the experimental data presented in Table 1. The nonlinear multiple regression model with high correlation coefficient (R²) and low relative error rate (RE %) has a high accuracy compared to linear multiple regression.

Table 1: Linear and nonlinear regression equations

<table>
<thead>
<tr>
<th>Relation</th>
<th>RE %</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{h}{P} = 1.505 \times \frac{k_w}{P} \times (\frac{P}{L})^{0.006} \times \alpha - 0.031 \times \left( \frac{P}{L} \right)^{-0.031} )</td>
<td>4.16</td>
<td>0.986</td>
</tr>
<tr>
<td>( \frac{h}{P} = 1.568 \times \frac{k_w}{P} \times (\frac{P}{L})^{0.104} \times \alpha^{-0.021} \times \left( \frac{P}{L} \right)^{0.003} )</td>
<td>4.09</td>
<td>0.987</td>
</tr>
</tbody>
</table>

As the angle increases, the relation between two parameters h/P and K/P becomes out of proportion and tends to linear relation. When the angle is 90 degrees, the geometry of the weir from the triangular plan to the weir of the channel is changed. Regarding angular data of 15 to 75 degrees, high precision regression relation was extracted between the three parameters h/P, K/P and \( \sin \alpha \) in the form of relation 8. In Eq. (8), \( \sin \alpha \) was used instead of \( \alpha \), where \( \alpha \) is in degrees.

\[
K_w = 0.66 \times \left( \frac{h}{P} \right)^{0.878} \times (\sin \alpha)^{0.04} \tag{8}
\]

By combining the two Eqs. (4) and (8), the equation 9 can be derived for the discharge coefficient of the triangular sharp-edged weirs plan in the following form.

\[
C_d = 0.568 \times \left( \frac{h}{P} \right)^{-0.183} \times (\sin \alpha)^{0.066} \tag{9}
\]

Given the value of the discharge coefficient of relation 9, it is possible to calculate the value of the discharge through the triangular sharp-edged weirs using the general relation of weirs (relation 10).

\[
Q = 2/3 \times 0.568 \times \left( \frac{h}{P} \right)^{-0.183} \times (\sin \alpha)^{0.066} \times \sqrt{2g \times L \times h^{3/2}} \tag{10}
\]

Table 2 compares the proposed relationship in the present study (Relation 10) with the numerical model and the experimental results in estimating the discharge of triangular sharp-edge overflows at angles of 15 and 45 degrees, respectively. As can be seen with a relative error of about 10%, the proposed equation (relation 10) can estimate the discharge.

![_table2.png](attachment:table2.png)

Table 2: Comparison between the results of the proposed relation in the present study with the numerical and laboratory models

<table>
<thead>
<tr>
<th>( n^* )</th>
<th>P (m)</th>
<th>h (m)</th>
<th>h/p</th>
<th>Experimental Q (L/t)</th>
<th>Numerical Q (L/t)</th>
<th>Equation 10 Q (L/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.092</td>
<td>0.0142</td>
<td>0.0737</td>
<td>1.972</td>
<td>2.201</td>
<td>2.014</td>
</tr>
<tr>
<td>15</td>
<td>0.092</td>
<td>0.0252</td>
<td>0.1059</td>
<td>3.263</td>
<td>3.588</td>
<td>3.169</td>
</tr>
<tr>
<td>15</td>
<td>0.092</td>
<td>0.0371</td>
<td>0.15</td>
<td>5.493</td>
<td>4.981</td>
<td>3.613</td>
</tr>
<tr>
<td>15</td>
<td>0.092</td>
<td>0.0525</td>
<td>0.197</td>
<td>8.271</td>
<td>7.8</td>
<td>9.484</td>
</tr>
<tr>
<td>15</td>
<td>0.092</td>
<td>0.069</td>
<td>0.237</td>
<td>10.296</td>
<td>11.75</td>
<td>12.21</td>
</tr>
<tr>
<td>45</td>
<td>0.103</td>
<td>0.0142</td>
<td>0.138</td>
<td>1.492</td>
<td>1.353</td>
<td>1.491</td>
</tr>
<tr>
<td>45</td>
<td>0.103</td>
<td>0.0252</td>
<td>0.245</td>
<td>3.468</td>
<td>3.1509</td>
<td>2.77</td>
</tr>
<tr>
<td>45</td>
<td>0.103</td>
<td>0.0371</td>
<td>0.36</td>
<td>6.085</td>
<td>3.895</td>
<td>5.277</td>
</tr>
<tr>
<td>45</td>
<td>0.103</td>
<td>0.0525</td>
<td>0.507</td>
<td>9.063</td>
<td>6.317</td>
<td>8.295</td>
</tr>
<tr>
<td>45</td>
<td>0.103</td>
<td>0.0691</td>
<td>0.6701</td>
<td>12.084</td>
<td>13.242</td>
<td>11.951</td>
</tr>
</tbody>
</table>

Conclusion

In the current study, using laboratory data, the discharge coefficient and the stage-discharge relationship in triangular sharp-edged weirs were investigated. Numerical simulation was also performed by Ansys-Fluent software. The performance of MR-Linear and MR-Nonlinear regression models on the application of the stage-discharge relation of triangular sharp-edged weirs was also investigated. Laboratory data were used for this purpose. The obtained values were compared with the models with the extraction relation of Distefano and Ferro [4] and the laboratory data of Kumar et al. [6] Also, a regression relation was derived for the discharge coefficient of triangular sharp-edged weirs using geometrical properties. This relation was compared with the extraction relations of other researchers and the results of the Fluent Numerical Model that give relatively good results. The discharge coefficient in triangular plan weirs can be increased about 1 to 8% (depending on the angle \( \alpha \)) relative to the suppressed weir, so these weirs can pass more discharge.

4. Reference