



As shown in these figures, a satisfactory match is observed between predicted and observed results.

#### 4- Conclusions

Comparison of predicted and observed results indicate that the proposed method and formulation can successfully be used in extending the model to include prediction of responses to cyclic loading of sands.

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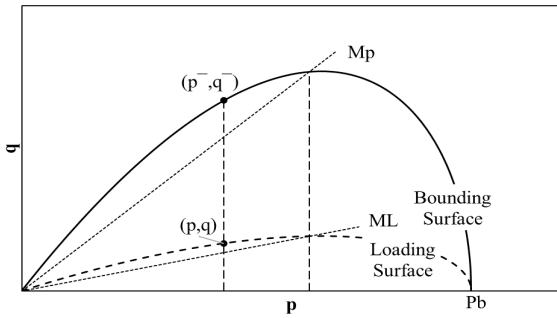
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model is used as bounding surface in the modified model and the loading surface is derived from the bounding surface using a deviatoric mapping rule as shown in Figure 1.



**Figure 1. Bounding surface, and determination of loading surface using the mapping rule employed**

Most model parameters such as the stress ratio  $q/p$  at peak of the yield surface,  $M_p$ , the stress ratio at phase transformation,  $M_{PT}$ , at which volumetric strains change from contractive to dilative, and stress ratio at failure,  $M_f$ , which is the highest stress ratio achievable by the soil at current state, are defined and determined exactly as in the original model. Elastic behavior is also assumed to be the same.

In the modified model, the following relationship proposed by Sheng et al. [6] is used to define the critical state line:

$$\ln e = \ln \Gamma - \lambda \ln \left( \frac{p}{p_a} + c_{cr} \right) \quad (1)$$

in which  $p$  and  $p_a$  are the current and atmospheric mean effective normal stress,  $e$  and  $\Gamma$  are the current void ratio, and its value at  $\frac{p}{p_a} + c_{cr} = 1$  and  $c_{cr}$  is a parameter that controls the curvature of the CSL in the  $e$ - $\log p$  plane. In the determination of the loading surface, the mapping is carried out by defining:

$$M_p = \frac{6 \sin \phi_p}{3 - (1 - 2t) \sin \phi_p} \quad (2)$$

in which  $t$  is a parameter that takes a value of zero in triaxial compression and 1 in triaxial extension. The stress ratio at the image point  $\bar{\eta}$  is determined using the following relationship:

$$\bar{\eta} = \alpha + \sqrt{M_\alpha^2 \left[ 1 - \left( \frac{p}{p_b} \right)^{0.5} \right]} \quad (3)$$

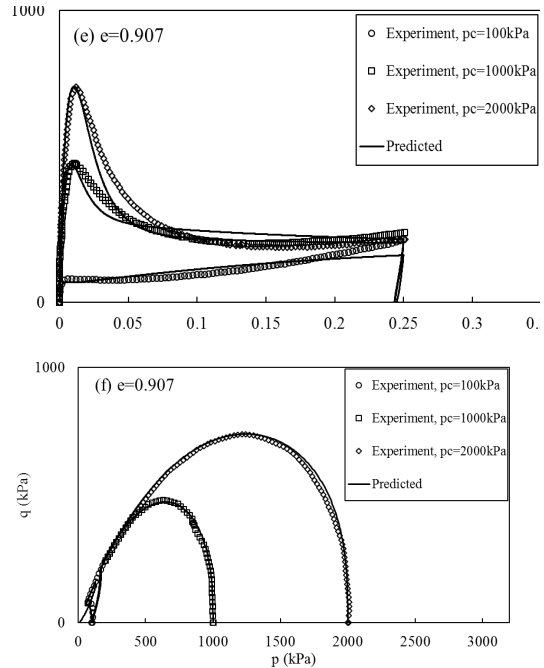
in which  $p_b$  is a parameter that controls the size of the bounding and loading surfaces,  $\alpha$  is the stress ratio at anisotropic consolidation and  $M_\alpha$  is the stress ratio at peak for anisotropically consolidated soils. Based on the above relationships, the stress ratio at peak for the loading surface,

$M_L$ , is defined as:

$$M_L = M_p \frac{\eta}{\bar{\eta}} \quad (4)$$

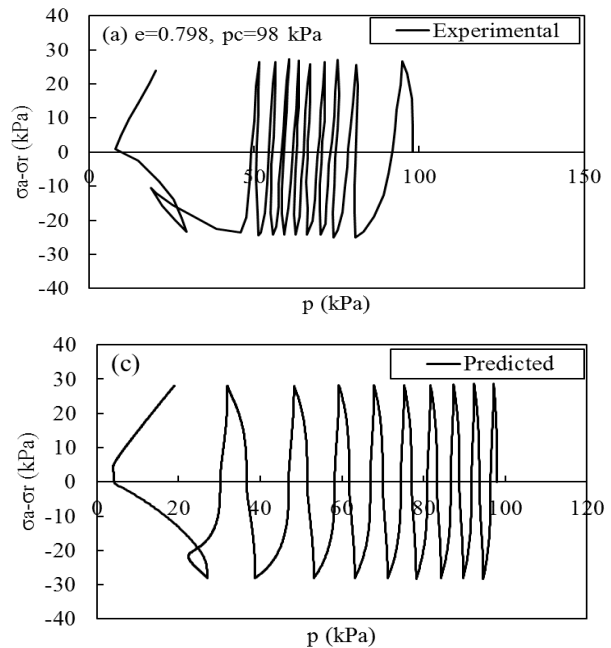
### 3- Results and Discussion

Results showing comparison of predicted and observed responses of sands are shown in Figure 2 for monotonic loading and in Figure 3 for cyclic loading of sands.



**Figure 2. Predicted and observed behavior of Toyoura sand in monotonic loading (data from [7])**

As shown in these figures, a satisfactory match is observed between predicted and observed results.



**Figure 3. Predicted and observed behavior of Toyoura sand in cyclic loading (data from [7])**



## Application of a Critical State Model for Cyclic Loading of Sands

Y. Javanmardi, S. M. R. Imam\*

Department of Civil and Environmental Engineering, Amirkabir University of Technology, Tehran, Iran

**ABSTRACT:** The purpose of the current paper is to extend a critical state constitutive model presented previously for sand behavior under monotonic loading such that it can predict behavior under cyclic loads as well. Due to the use of the critical state soil mechanics framework, the original model was able to predict sand behavior over a wide range void ratios and confining pressures, and take into account various aspects of behavior of loose and dense sands, including inherent and stress-induced anisotropies, softening and liquefaction of sands under monotonic loads. Extension of the base model for cyclic loading is accomplished through the use of bounding surface plasticity. Yield surface of the original model is used as the bounding surface of the new model, and also its loading surface using a deviatoric mapping rule. A new hardening modulus is used that enables predicting the behavior during loading and unloading. Flow rule of the original model is also modified in order to enable better prediction of the loading-unloading behavior, especially after phase transformation. Predictions based on this model showed satisfactory match with observed behavior of sands over a wide range of void ratios and confining pressures in drained and undrained monotonic and cyclic loading.

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### 1- Introduction

The general framework of critical state soil mechanics (CSSM) was first introduced and used by Roscoe et al. [1], and Schofield and Ruth [2] for modeling the behavior of remolded clays and led to the development of the Cam Clay model, which has been used relatively successfully for predicting the behavior of clays for decades. However, use of this framework was not as helpful in modeling sand behavior, mainly due to the complexity in the behavior of sands, in which effects of density and confining pressures on the dilatancy and other aspects of behavior should both be taken into account simultaneously.

A major advantage of constitutive models that are presented in the framework of CSSM is that they do not require calibration and parameter determination when behavior of soils at various void ratios and confining pressures is to be predicted. A single set of model parameters is used in such models and soils having different states (void ratios and confining pressures) do not have to be treated as different materials [3, 4]. A number of models have been presented for sands in this framework over the past two decades, mainly for monotonic loading but modeling for cyclic loading has been limited [4, 5].

The purpose of the current papers is to extend the model presented by Imam et al. [5], a critical state constitutive model presented for sand behavior under monotonic loading,

such that it can predict behavior under cyclic loads as well. Due to the use of critical state soil mechanics framework, the original model was able to predict sand behavior over a wide range void ratios and confining pressures, and take into account various aspects of behavior of loose and dense sands including inherent and stress-induced anisotropies, and softening and liquefaction of sands under monotonic loads.

### 2- Methodology

Extension of the base model for cyclic loading is accomplished through the use of bounding surface plasticity. Yield surface of the original model is used as the bounding surface of the new model and a mapping rule based on deviatoric stresses is employed in order to define the loading surface using yield surface of the original model. A new hardening modulus is used that enables predicting the behavior during loading and unloading. Flow rule of the original model is also modified in order to enable better prediction of the loading-unloading behavior, especially after the state of phase transformation is reached. A total of fourteen model parameters are used in the modified model for predicting sand behavior under both monotonic and cyclic loading. Formulation of the modified model is presented and predictions based on this model are compared with experimental results obtained from tests conducted on two types of sands. Comparisons of predicted and observed behaviors of these sands over a wide range of void ratios and confining pressures, in drained and undrained loading, and also under monotonic and cyclic loads.

Details of formulation and performance of the base model can be found in Imam et al. [5]. Yield surface of the original

Corresponding author, E-mail: rimam@aut.ac.ir